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Temperature impact on stochastic mortality modelling in Italy

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Abstract

Environmental changes have been shown to influence human health and mortality over time. In this thesis, the relationship between trends in mortality and trends in temperature changes (highest, average and lowest), as a proxy of changes in the climate, is analysed by using annual data for specified periods of the year (warm months, cold months, and all of the months) and for three areas of Italy: the North, Centre and South. Furthermore, the temperature-related stochastic mortality model recently proposed by M. Seklecka, A.A. Pantelous and C. O'Hare[1] is fitted with Italian data and investigated in-depth, discussed and compared with other models. The authors have noted improvements in fitting and forecasting processes, mostly for cold months, by fitting the model with UK mortality and average temperature.

This thesis seeks to discover whether that model may provide accurate results even for southern European populations, which live in a milder climate, and whether the highest or lowest temperature may be more significant than the average temperature for constructing projected life tables.

The findings confirm that the proposed model might provide improvements in modelling human mortality rates and, as a novelty in the literature, that considering the highest or the lowest temperature results to be much more significant than implementing the average temperature or dividing the year into three periods. Nevertheless, several inconsistencies in the findings and a few unsteady results suggest that a different implementation of the temperature-related factor might be more appropriate and should be further explored in additional studies.

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[1] Seklecka M, Pantelous AA, O'hare C. Mortality effects of temperature changes in the United Kingdom. *Journal of Forecasting*. 2017; **36**: 824-841. https://doi.org/10.1002/for.2473.

Abstract

Molte ricerche hanno dimostrato come i cambiamenti ambientali influenzino la salute e la mortalità dell'essere umano. In questa tesi, è investigata la relazione nel tempo tra la mortalità e le temperature massime, medie e minime, come proxy climatiche. L'analisi è effettuata per il Nord, il Centro ed il Sud Italia, utilizzando dati annuali riferiti ai soli mesi caldi, quelli freddi, o all'intero anno.

Inoltre, il modello stocastico di mortalità recentemente introdotto da M. Seklecka, A.A. Pantelous e C. O'Hare[2], che incorpora al suo interno un fattore correlato alla temperatura, è fittato sui dati italiani ed investigato, discusso e comparato con altri modelli noti in letteratura. Gli autori, testando il modello per la mortalità e la temperatura media del Regno Unito, hanno notato miglioramenti nel fitting e nella previsione, soprattutto per i mesi freddi.

La seguente trattazione si pone due obiettivi principali, il primo è scoprire se risultati altrettanto positivi si riescano a replicare anche per popolazioni, come quelle dei paesi dell'Europa meridionale, caratterizzate da un clima più mite; il secondo è comprendere se le temperature massime o quelle minime possano essere più significative, rispetto a quelle medie, per costruire tavole di mortalità proiettate.

I risultati confermano che un modello correlato alla temperatura potrebbe fornire miglioramenti nel modellizzare la mortalità umana. Inoltre, come assoluta novità in letteratura, mostrano che la temperatura massima e quella minima sono più indicative che quella media per questa ricerca. Infine, evidenziano come la suddivisione dell'anno in tre sotto-periodi si sia rivelata non essenziale. Ciononostante, alcune incongruenze e alcuni risultati poco robusti nel corso della trattazione, suggeriscono che un'implementazione differente della temperatura nel modello potrebbe risultare più appropriata, e che sarebbe opportuno approfondire questa analisi in studi futuri.

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[2] Seklecka M, Pantelous AA, O'hare C. Mortality effects of temperature changes in the United Kingdom. *Journal of Forecasting*. 2017; **36**: 824-841. https://doi.org/10.1002/for.2473.

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Chapter 1

Introduction

The interest paid to the mortality risk is continuously increasing, since governments, individuals and life insurers must address it in order to avoid costly evaluation errors which may lead to substantial losses. Hence, it is crucial that mortality models be the most accurate possible, since they return the technical bases used to calculate financial instruments linked to human life.

Over the last 100 to 200 years, the increase in life expectancy has improved incredibly compared to the results which emerged up until the end of the 18^{th} and beginning of the 19^{th} century, exhibiting a rise of 30 years of life expectancy at birth during this period, mostly owing to the extraordinary medical and hygienic progresses available. Hence, predicting what might happen in the future is very difficult nowadays.

Many studies indicate how temperature – and climate in general, which is a direct index of the Earth's health – may affect both the quality and longevity of life. Every year, many people die because of weather conditions, not only directly from phenomena such as extreme temperatures, droughts, floods or storms, but also indirectly: climate changes are expected to cause an annual worldwide increase in deaths of about 250,000 persons from 2030 to 2050 as a result of malnutrition, diarrhoea, malaria and heat stress (*World Health Organization, 2014*). Naturally, each country faces a distinct climate, with different economic resources and a different culture. Some studies show that, in various populations, the relationship between climate and mortality exhibits different absolute values (*Gasparrini et al., 2015; Analitis et al., 2008; Meehl and Tebaldi, 2004*). Furthermore, as it might be expected, older adults are the most vulnerable to climatic changes (*Hajat et al., 2014*). Finally, the absence of adaptation to climatic changes is revealed to be a noticeable factor which contributes to the temperature-linked mortality (*Patz et al., 2005; McMichael et al., 2006*)[3].

Since the 17th century, several mortality models have been proposed in the literature, with the aim of providing the best evaluation of mortality rates. The first models reflected simple mathematical law, while, starting with the Lee & Carter model (*1992*), later models also included the application of stochastic factors to take into consideration the variability surrounding the estimation. Many extensions of the Lee & Carter model have been introduced in the last 20 years, sometimes showing accurate results. Recently, several authors have attempted to give an economic interpretation to the Lee & Carter latent factor $\boldsymbol{k_t^1}$ or to explain mortality dinamics through macroeconomic fluctuations (see, for example, *J.A. Tapia Granados, 2008, 2011; K. Hanewald, 2011; Niu G. and Melenberg B., 2014*).

Nonetheless, none of them has ever got a temperature-related factor before the article published by Secklecka, Pantelous and O'Hare, "Mortality effect of temperature changes in the United Kingdom", in 2017.

Following this paper, this thesis investigates the relationship between trends in temperature changes and trends in mortality in three regions of Italy (North, Centre and South). The motivation behind the choice of dividing the study into three areas, instead of considering the entire country at once, derives from the climatic heterogeneity of Italy. Furthermore, the analysis examines the highest, average and lowest temperature changes in order to look for a possible difference in the use of these series.

This investigation begins in Chapter 2, where the longevity risk is explained within the Solvency II framework to determine the general issue.

Then, in Chapter 3, several mortality laws and stochastic mortality models are briefly presented, mostly focusing on those which are more helpful for the subsequent analysis.

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[3] As Darwinian theory has taught, "It is not the most intellectual of the species that survives; it is not the strongest that survives; but the species that survives is the one that is able best to adapt and adjust to the changing environment in which it finds itself." Cit. Leon C. Megginson.

Italian mortality and regional temperature data are introduced in Chapter 4 with the help of graphs to understand the underlying heterogeneity.

In Chapter 5, the series of time-dependent Lee & Carter model (1992) factors $\boldsymbol{k_t^1}$, and the series of the highest, average, and lowest temperatures for each gender, for each region and for the age range of 20-85 years, are studied through the Phillip-Perron (*1988*) and the Kwiatkowski-Phillips-Schmidt-Shin (*1992*) tests. Subsequently, the statistical association between $\boldsymbol{k_t^1}$ and temperature is examined by calculating the Pearson, the Kendall and the Spearman correlation coefficient, testing the null hypothesis of no correlation for the Pearson's. A survey of the Pearson correlation coefficients between the central mortality rates and the three temperature series is also conducted, and it is checked the robustness of the results, since this correlation coefficient will be directly used in the temperature-related model. The results justify the introduction of the model and the continuation of the analysis.

As a result, the Secklecka, Pantelous and O'Hare temperature-related factor model is proposed in the Chapter 6. First, the model is presented, its factors are singularly explained, and the estimation of its parameters is discussed before attempting to fit them. Then the parametric and projection risks are investigated. This first risk is monitored by analysing the parameter uncertainty, using the bootstrapping technique. The second risk is examined by exploring the goodness of fit: first by scanning the analysis of the residuals, then by calculating several error measures and by checking their robustness, and finally by examining the same measures for the out-of-sample data. Once completed, the model has been forecasted. All of these steps are continuously performed, comparing the results with those of the Lee and Carter (*1992*), the Plat (*2009*), and the O'Hare and Li (*2012*) models.

Lastly, Chapter 7 shows the conclusion of this work.

Chapter 2

Longevity Risk

A good assessment of the mortality rates is critical for insurance (and reinsurance) companies, pension funds and governments. The longevity risk is profoundly affected by an incorrect valuation of the expected mortality rates and it must be adequately controlled in order to prevent incurring substantial losses. It is a sub-risk of a more general demographic risk, which also includes the insurance risk. Insurance risk is a pooling risk; it is derived from the random deviation of the number of deaths from their expected value, and it can be reduced through diversification, i.e. increasing the number of policies in the portfolio. On the contrary, longevity risk is a systematic risk; it emerges when the observed mortality rates are consistently different from those expected, and it may not be reduced through diversification because it moves in the same direction for all policies. Another classification for the demographic risk is that it can be individual, which is the possibility that a policyholder lives longer than expected by the policy, or aggregated when the average number of years of policyholders' lives in the entire portfolio is taken into consideration. In the case of a pure endowment or a pension plan, it is also crucial to neither to incur massive losses, by overestimating the mortality death rates, nor to miss profits by underestimating them. In the first case, the payment period and the liabilities would increase; in the second case, insurance companies, aside from the fact that they would suffer from competition with other companies, need to save extra amounts to cover actuarial liabilities, which would immobilise these funds and eliminate the possibility of using them for other assets. Obviously, the opposite is true for life insurance companies.

European insurance companies must fulfil a strict obligation of adequacy in their mortality profile characterisation; in fact, according to the principles of the International Accounting Standards Board (IASB), every source of risk must be measured at its fair value in order to enter it in the budget.

2.1 Longevity risk in Solvency II[4]

Solvency II is the current regulatory framework for European insurance and reinsurance companies. Its purpose is to increase the level of harmonisations of solvency regulations. Solvency II is divided into three pillars, following what Basel II (now updated to Basel III) represents in banking prudence regulation. Solvency II is founded by an integrated risk analysis which requires calculating the market-consistent value of assets and liabilities by observing the market prices (mark-to-market), where possible, or by using a mark-to-model technique if it is not.

The technical provisions must be evaluated as the sum of the best estimate liability and risk margin. The calculation should be segmented by homogeneous product type. The best estimate liability (BEL) is the present value of expected future cash flows, discounted using a risk-free yield curve. The risk margin (RM) is intended to increase the technical provisions to the amount that would have to be paid to another insurance company in order for them to take on the best estimate liability.

The Minimum Capital Requirement MCR is the minimum level of security below which the amount of financial resources should not fall. It is defined as a simple factor-based linear formula which is targeted at a Value at Risk measure over one year with 85% confidence.

The Solvency Capital Requirement (SCR) should reflect a level of eligible own funds that enable insurance and reinsurance undertakings to absorb significant losses and that give reasonable assurance to policyholders and beneficiaries that payments will be made as they fall due. The SCR is a Value at

[4] This section quotes many definitions from the Solvency II Directive (Directive 2009/138/EC).

Risk measure based on a 99.5% confidence interval of the variation over one year of the amount of basic own funds.

The SCR has to cover some prescribed risk such as:

- market risk
- life underwriting risk
- non-life underwriting risk
- health underwriting risk
- counterparty default risk
- operational risk

The SCR may be calculated by using an internal model, which is subject to authorisation by the supervisory authority of the insurance companies, or by using the standard formula. In the second case, the SCR for each risk is calculated through standard prescribed stress tests or factors and then aggregated by using correlation matrices in sub-modules and modules. The overall SCR is obtained by aggregating the risk modules through other correlation matrices.

The longevity risk is one of the sub-modules of the life underwriting risk, and also of the health underwriting risk. This was adequately explained by the European Insurance and Occupational Pensions Authority (EIOPA) in 2014:

The stress factor for longevity risk is intended to reflect the uncertainty in mortality parameters as a result of mis-estimation and/or changes in the level, trend and volatility of mortality rates and captures the risk of policyholders living longer than anticipated.

The underlying assumptions for the longevity risk sub-module may be summarised as follows:

· The annual mortality improvements follow a normal distribution.

· For the simplified calculation of the capital requirement for longevity risk it is assumed that the average age of policyholders within the portfolio is 60 years or more.

· It is furthermore assumed that the average mortality rate of the respective insured persons does not increase by more than 10% each year.

2.2 Current situation

By analysing the mortality trends of the last decades through the survival function (see Fig. 2.1), the rectangularisation and the expansion of its general shape is visible. The rectangularisation of the survival function indicates a higher concentration of the probability distribution around the Lexis point and its approach to the maximum lifespan. The expansion means that the maximum lifespan is rising. The consequence is an increasing life expectancy.

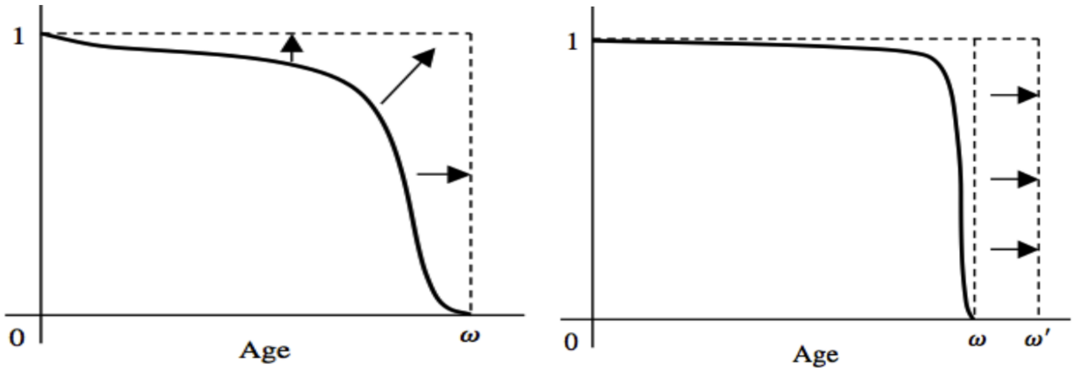

Fig. 2.1 - Rectangularisation and expansion of the survival curve (S. Levantesi, "Laboratorio di tecnica attuariale").

Exploring the example of the country of Italy, the average life expectancy has increased by 20 years from the second half of the 19th century, and is now growing faster. The Italian National Institute of Statistics (ISTAT) expects that in 2060 the life expectancy at birth will be 85.5 years for Italian males and 90.3 years for Italian females, while in 2015 it was 81.1 years for males and 86 years for females. Figure 2.2 shows the Italian data from 1931 to 2006.

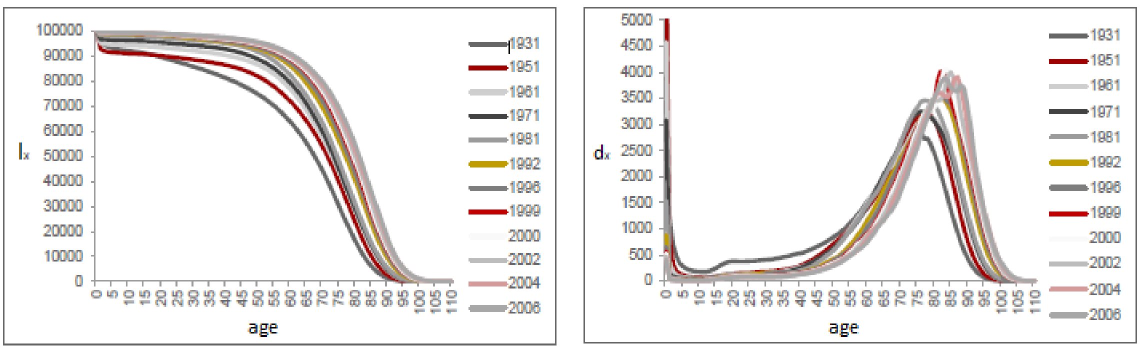

Fig. 2.2 - Survival and death curve, Italian data (S. Levantesi, "Laboratorio di tecnica attuariale").

2.3 Risk Management

Operations of portfolio diversification are not useful for minimising the longevity risk due to its systematic nature. Therefore, there are two possibilities to deal with that risk – transferrance to third parties, and proper management.

Transferring longevity risk to third parties means to apply either traditional reinsurance or financial reinsurance, but missed profits, and sometimes even losses, might be incurred in both cases. Traditional reinsurance does not often work properly to reduce the longevity risk, because the price of the reinsurance may be much higher for the insurer to erase all the profits. That issue directly derives from the non-pooling nature of the longevity risk, which increases the reinsurer’s exposure for every type of treaty. Likewise, appealing to the financial market through securitisation does not give the desired answers, although it may have the necessary amounts to decrease the longevity risk owing to the larger volume of financial market exchanges and non-correlation with other financial instruments. Without going into too much detail, it is worth remembering that Longevity Swap has not been revealed to be a convenient contract for the counterparties, whereas Longevity Bonds have been more interesting to the market and do not enhance the credit risk. Nevertheless, these solutions alone are not sufficient.

The second possibility for dealing with longevity risk is by managing it directly; this basically means that insurance companies implement projected mortality tables, provided by using statistical procedures, which guarantee a better evaluation of the mortality profile. In this manner, they attempt to capture the correct actuarial value of the amounts to correspond to the annuitants beforehand.

There are three different methods which may be used to project mortality rates: the use of expectation models, explanatory models or extrapolative models.

The first method refers to the field of expert actuaries and demographers, who use their knowledge to develop opinions for the future mortality trend.

The second method, the use of explanatory models, considers exogenous risk factors, e.g. socio-economic, biomedical or environmental data, in modelling mortality. Although good improvements in fitting quality have been recently discovered, the forecast horizon of these models remains too short to provide good predictions, and the underlying relationship between mortality and risk factors is not always easy to capture.

The last method – extrapolative models – currently represents the models mainly used by actuaries and governments. These models develop the projection by observing historical mortality trends and patterns, which provide a dynamic mortality rate as a function of the calendar year and age. These models implicitly assume that the past path will similarly continue in the future, they are easy to fit and to forecast for an extended horizon, and they give fine results when the past reference period is accurately chosen. These models may be divided into deterministic and stochastic versions.

Deterministic extrapolative models merely extend the past mortality trends to the future. Though the mortality rates are well evaluated compared to the ones found through a hypothesis of static mortality, they still do not consider that the projection itself is affected by uncertainty (Fig. 2.3).

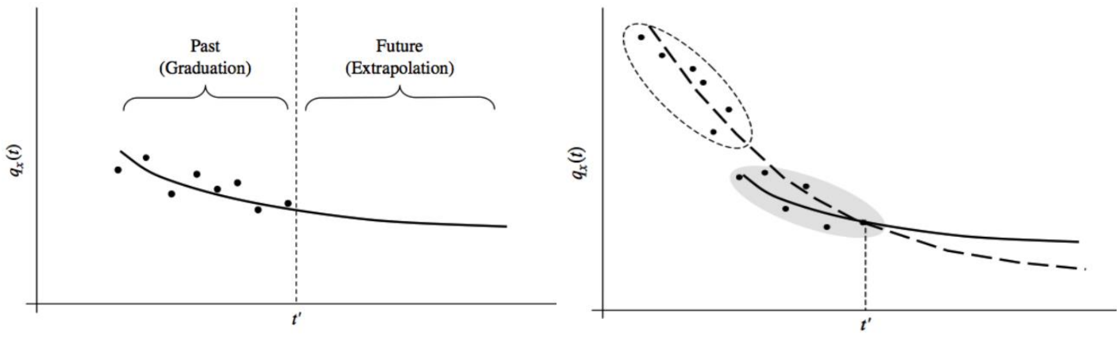

Fig.2.3 – Deterministic extrapolative method (Pitacco, Denuit, Haberman, Olivieri A. (2009), "Modelling Longevity Dynamics for Pensions and Annuity Business").

In fact, three types of risk must be controlled:

1) Projection risk: This risk derives from the inadequacy of the model to evaluate the mortality trend.

2) Parametric risk: This risk is inherent in the uncertainty of the parametric values of the model.
3) Process risk: This risk is caused by the stochastic nature of the process which controls mortality. It is the risk of random fluctuation.

The first two risks give the uncertainty risk, which may be reduced using proper stochastic models. The third risk may be reduced by raising the portfolio dimension or through reinsurance when possible.

The stochastic extrapolative methods, on the contrary, make their projections based on a probability distribution. In this way, even the uncertainty risk of the projection is calculated by giving an interval estimation (Fig. 2.4).

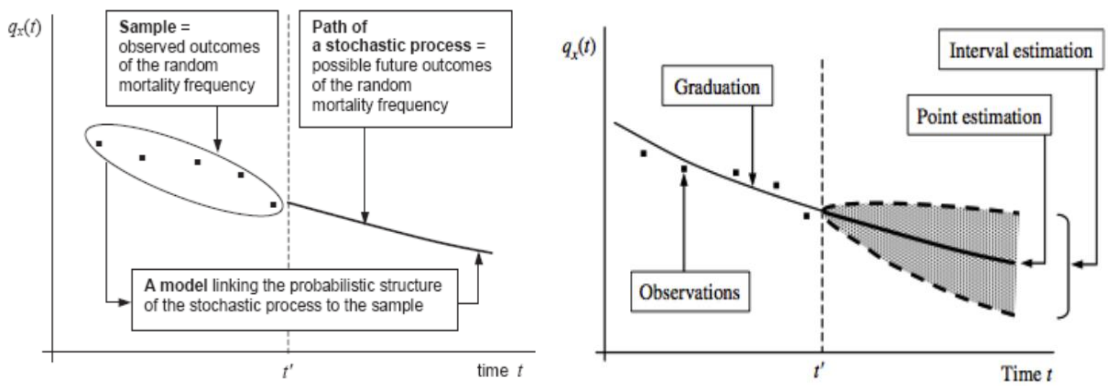

Fig.2.4 – Stochastic extrapolative method (Pitacco, Denuit, Haberman, Olivieri A. (2009), "Modelling Longevity Dynamics for Pensions and Annuity Business").

The stochastic extrapolative methods may evaluate both the random and the systematic mortality fluctuation and may assign probabilities to discrete or continuous scenarios. The main models used in the literature are:

1) Econometric models
2) Models founded on interdependent mortality projection at specific ages
3) Models which use evaluation procedures on standard time series

The model analysed in this thesis may be interpreted as a combination of a stochastic extrapolative model which uses evaluation procedures on standard time series and an explanatory method, since it includes a temperature-related factor.

Ultimately, there are several models in the literature (even one based on subjective expectation (*Booth et al., 2008*); each one has positive and negative aspects and sometimes it is not easy to state which one is better. Besides, changing some assumptions or calculating different error measures may provide different rankings.

Still, in 2007 the Continuous Mortality Investigation – Life Office Mortality Committee set a number of desirable objectives for projection models:

- Easy to use
- Easily-to-understand parameters
- Parsimony but good adherence to data
- Able to reflect the cohort effect
- Reasonable best estimate projection
- Presence of confidence intervals
- Ability to generate simple path

Nonetheless, accurate quantification of the mortality table is required because some components of uncertainty remain for the future projection:

- Model uncertainty: The "correct" underlying model is unknown.
- Parameters uncertainty: Parameters are estimated from a finite dataset.
- Stochastic uncertainty: This reflects the random fluctuation.
- Measure and heterogeneity.
- The experience might not be a good proxy for a future one.

A general historical review of the mortality models is presented in the next chapter, and the Lee & Carter *(1992)*, the Plat *(2009)* and the O'Hare and Li *(2012)* stochastic models are further examined, since these will be compared to the temperature-factor related model introduced in the Chapter 6.

Chapter 3

Mortality models

This chapter aims to introduce the mortality models as they have been proposed over time. Some of the most important mortality laws are described in the first section, from the beginning of the 17th century to the present day. The study of the Lee & Carter *(1992)*, the Plat *(2009)* and the O'Hare and Li *(2012)* stochastic models is deepened in sections 3.2, 3.3 and 3.4, respectively, since they will be subsequently analysed in comparison to the temperature-related model. Finally, the CBD and the Renshaw-Haberman stochastic models, well-known in mortality literature, are proposed in section 3.5.

3.1 Mortality laws

Over the last three centuries, many mortality laws have been proposed to try to describe mortality shape using a mathematical function. Some of the most representative are proposed below, in chronological order.

De Moivre *(1725)* considered that the survival function S_x was linear:

$$S_x = \{1 - \frac{x}{\omega} \qquad 0 \leq x \leq \omega;\ x > \omega \tag{1}$$

Where ω is the maximum age (ω=86 for De Moivre). The main problem is that this model is not very realistic. Generalisations of this model were presented by De Graaf *(1729)* and Babbage *(1823)*.

According to Gompertz *(1825)*, mortality exponentially rises with age; he introduced what is currently called the "force" of mortality:

$$\mu_x = \alpha\, e^{\beta x} \qquad \alpha > 0 \tag{2}$$

Makeham *(1860)* generalised the Gompertz mortality law by introducing a constant addend which for the first time took into consideration accidental causes of deaths, supposedly independent of the natural ageing:

$$\mu_x = \gamma + \alpha\, e^{\beta x} \qquad \alpha,\beta > 0\ ;\ \gamma \geq 0 \tag{3}$$

Both the Gompertz and the Gompertz-Makeham models give good results for middle ages, but they capture neither the high mortality at the earliest ages nor the bump in the mortality curve around the range of ages from 18 to 25. Some extensions, such as in the Lazarus *(1867)* and the Thiele *(1867)* models, have been proposed to overcome these problems. The features of these laws lead us to the Heligman and Pollard *(1980)* model, which will be described soon.

But first, it is interesting to show the Lexis *(1878)* model, in which the author hypothesises a Gaussian distribution of the age at death:

$$f_0(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\bar{x})^2}{2\sigma^2}} \qquad x \geq x' \tag{4}$$

Where $\bar{x}$ is the Lexis point and x' is the minimum applicable age.

The Weibull *(1939)* law for the force of mortality is mentioned, too, because of its widespread use in the reliability theory:

$$\mu_x = \frac{\alpha}{\beta}\ \left(\frac{x-\vartheta}{\beta}\right)^{\alpha-1} \qquad x \geq \vartheta \tag{5}$$

Where α, β and ϑ are positive parameters of shape, scale and localisation.

Eventually, the Heligman and Pollard *(1980)* model was developed, which refers to the odd ratio between death and life probabilities. It consists of three addends whose roles are to capture infantile, accidental and senile mortality:

$$\frac{q_x}{p_x} = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^{\ 2}} + GH^x \tag{6}$$

Where ***A, B, C, D, E, F, G, H*** are parameters to be estimated. This model offers a proper fitting for the entire span of life, and it is used for some applications yet (see Fig. 3.1).

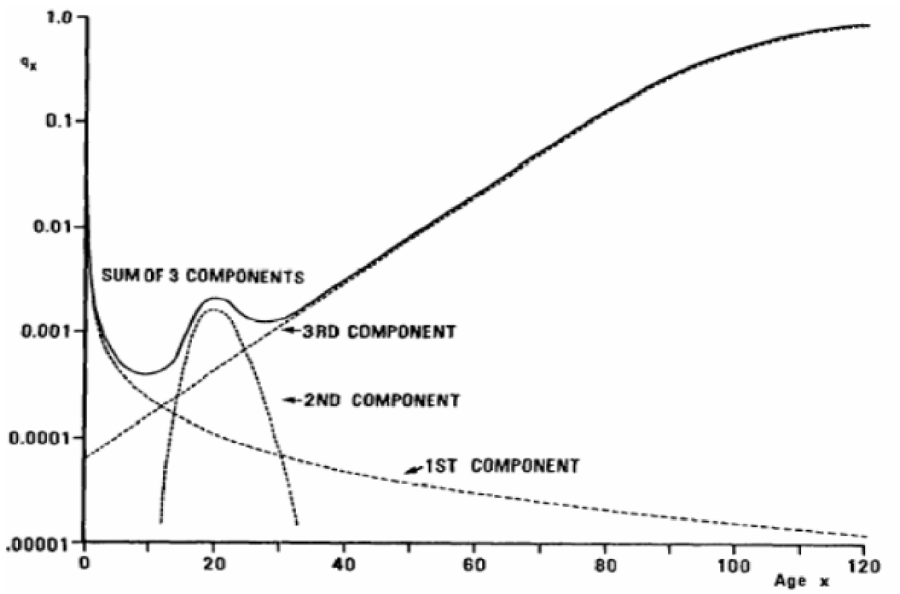

Fig. 3.1 - L. Heligman, J.H. Pollard, "The Age pattern of Mortality", Journal of the Institute of Actuaries, 1980; **107**: 49-80.

3.2 Lee and Carter

The Lee & Carter model (hereafter referred as the LC model) was introduced by Ronald Lee and Lawrence Carter in 1992 (Lee, R. and Carter, L. "Modeling and Forecasting U.S. Mortality"; 1992, *Journal of the American Statistical Association* **87**: 659-671). The LC model is the most famous and widely used stochastic mortality model. By observing previous mortality rates, it uses time series to extrapolate the time trend through a one-factor stochastic model. Due to its endogenous mechanism of parameter calculation year by year, it manages to capture changes in the mortality trend rather well.

The central mortality rates have a log-bilinear shape:

$$\boldsymbol{ln\,(m_{x,t}) = b_x^1 + b_x^2 k_t^1 + \varepsilon_{x,t}} \tag{7}$$

Where:

$\boldsymbol{m_{x,t}}$ is the central mortality rate of people aged $\boldsymbol{x}$ in the year $\boldsymbol{t}$. It is calculated as the ratio between the number of deceased people and the exposed to the risk both for age $\boldsymbol{x}$ and year $\boldsymbol{t}$: $\boldsymbol{m_{x,t}} = \frac{\boldsymbol{Dx,t}}{\boldsymbol{Ex,t}}$.

$\boldsymbol{b_x^1}$ describes the average behaviour of the central mortality rate for every age. In addition, it ensures that the shape of the mortality curve conforms to the experience. By using the classic constraints (see Par. 3.2.1), this non-parametric term represents the arithmetic mean of the $\boldsymbol{ln}\,(\boldsymbol{m_{x,t}})$ over all the observed period.

$\boldsymbol{b_x^2}$ is another non-parametric term, which explain how $\boldsymbol{ln}(\boldsymbol{m_{x,t}})$ reacts to the passage of time. It is a sensitivity parameter of the velocity of the mortality rate's response to $\boldsymbol{k_t^1}$ for every age: $\frac{\boldsymbol{dln(m_{x,t})}}{\boldsymbol{dt}} = \frac{\boldsymbol{b_x^2 dk}}{\boldsymbol{dt}}$.

$\boldsymbol{k_t^1}$ represents a mortality changes index over time[5].

$\boldsymbol{\varepsilon_{x,t}}$ indicates the error term, the effects not captured by the model. The errors are assumed as i.i.d. with Normal distribution $\boldsymbol{(0,\sigma^2_\varepsilon)}$ $(\boldsymbol{\sigma^2_\varepsilon<\infty})$.

This model is fitted with Italian data, and its parameters are plotted in Figure 3.3 to give a broader view of them[6]. The exponential trend shape of $\boldsymbol{m_{x,t}}$ agrees with the opinion, currently verified, that the life expectancy increases over time, but less quickly year by year[7]. The central mortality rate is modelled as:

$$\boldsymbol{m_{x,t} = exp\,(b_x^1 + b_x^2 k_t^1 + \varepsilon_{x,t})} \tag{8}$$

The strongest points of the LC model are its simplicity and good fitting results. Nonetheless, it has two principal negative aspects. The first one is that it does not hold a cohort factor, although the evidence confirms that it should be

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[5] The parameters meaning may be easily understood by viewing their plots (see Fig. 3.3 or Fig. 4.1).

[6] The data will be presented in Chapter 4. It is used the Brohuns et al. proposal shown in section 3.2.3.

[7] The forecast of $\boldsymbol{m_{x,t}}$ for the next 50 years is shown in Fig. 4.3 (a), (b) and (c) in section 4.1.

added (see Fig. 3.2). The second negative aspect is that it does not consider, according to the authors' version, the mortality rates' heterogeneity at different ages; despite the fact that that it is remarkable for the youngest persons and mostly for the eldest (see section 3.2.3).

3.2.1 Estimation of parameters

The LC model as described in equation (7) is underestimated. In fact, the terms of the second member are not observable. There is no unique parameterisation of the model, so some restrictions are required. The authors have applied these two constraints:

$$\begin{cases} \sum_x \boldsymbol{b_x^2} = \mathbf{1} \\ \sum_t \boldsymbol{k_t^1} = \mathbf{0} \end{cases} \tag{9}$$

Therefore, the $\boldsymbol{b_x^1}$ parameter results to be the temporal mean of the logarithm of the central mortality rates for every age $\boldsymbol{x}$:

$$\widehat{\boldsymbol{b_x^1}} = \frac{\mathbf{1}}{\boldsymbol{T}} \sum_t \boldsymbol{ln(m_{x,t})} \tag{10}$$

In order to calculate the optimal solution, the authors used Singular Value Decomposition (SVD), which is mathematically equivalent to performing the Principal Component Analysis (PCA) of the log-mortality rates' covariance matrix. However, the $\widehat{\boldsymbol{b_x^2}}$ and $\widehat{\boldsymbol{k_t^1}}$ estimations do not ensure that the number of estimated deaths is similar to those observed. Hence, a second-stage estimation is required, which is a second iterative procedure which, by fixing $\widehat{\boldsymbol{b_x^1}}$ and $\widehat{\boldsymbol{b_x^2}}$, returns the $\widehat{\boldsymbol{k_t^1}}$ value which satisfies the condition $\widehat{\boldsymbol{D_t}} = \boldsymbol{D_t}$. This equation may have no single solution or may not present any solution, so the model can become inconsistent. By using the Brouhns et al. (*2002*) variant instead, the parameters may be determined by maximising the log-likelihood function based on the model (see equation (17) and section 3.2.3).

3.2.2 Forecasting

The $\boldsymbol{k_t^1}$ mortality index is the only parameter to forecast, since $\boldsymbol{b_x^1}$ and $\boldsymbol{b_x^2}$ are constant over time. The mortality index $\boldsymbol{k_t^1}$ is modelled and projected as a stochastic time series by using an appropriate AutoRegressive Integrated Moving Average (ARIMA) process found by the Box-Jenkins procedure. Lee and Carter, from their application to U.S. mortality, obtained an ARIMA (0,1,0) linear-trend for the latent factor $\boldsymbol{k_t^1}$. Thus, the $\boldsymbol{k_t^1}$ results are not independent from one another, but their innovations are. Therefore, it behaves as a simple random walk with drift:

$$\boldsymbol{k_t^1} = \boldsymbol{k_{t-1}^1} + \boldsymbol{d} + \boldsymbol{e_t} \tag{11}$$

Where:

$\boldsymbol{d}$ is the drift term.

$\boldsymbol{e_t}$ is the error term, supposed Normal $(0,\sigma^2_e)$ distributed ($\sigma^2_e<\infty$).

The $\boldsymbol{k_t^1}$ standard error positively depends on the forecast horizon period $\boldsymbol{s}$:

$$\boldsymbol{\sigma_s} = \boldsymbol{\sigma_1}\sqrt{\boldsymbol{s}} \tag{12}$$

Finally, it is possible to easily obtain $\boldsymbol{m_{x,t}}$ for every age $\boldsymbol{x}$ and year $\boldsymbol{t}$.

3.2.3 The Brouhns et al. (*2002*) proposal

One of the main negative aspects of the LC model is that it implicitly assumes that the random errors $\boldsymbol{\varepsilon_{x,t}}$ are homoscedastic, i.e. they hold the same variance among ages.

That assumption is not very realistic, especially for older ages, where the small number of deaths produces statistical problems. Moreover, older adults are the most significant to study since most of the financial products linked to mortality refer to them. In order to find a remedy, N. Brouhns, M. Denuit and

J.K. Vermunt (*2002*) proposed a modification of the LC model, where the deaths are distributed as a Poisson distribution:

$$\boldsymbol{ln(m_{x,t}) = b_x^1 + b_x^2 k_t^1} \tag{13}$$

$$\boldsymbol{D_{x,t} \sim Poisson\ (E_{x,t}\ \mu_{x,t})} \tag{14}$$

Where $E_{x,t}$ is the number of the exposed to risk and $\mu_{x,t}$ is the force of mortality. Therefore, a Poisson random variation of deaths confers heteroscedasticity to the error term and more realism to the model.

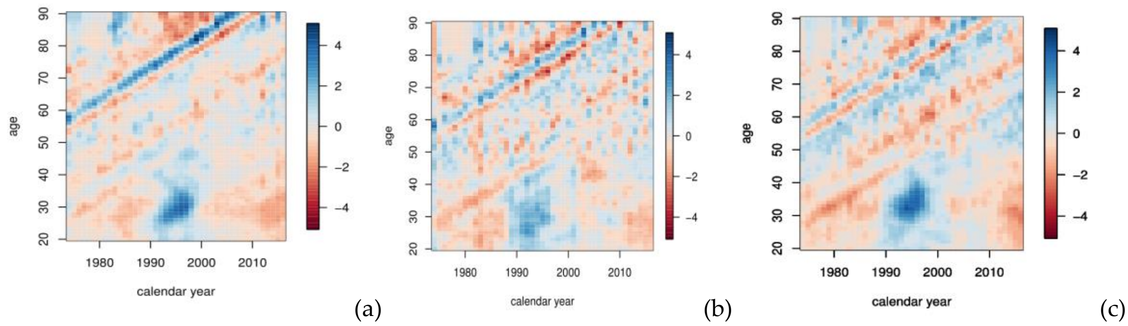

Fig. 3.2 – LC heat map of the residuals for: (a) females from Lombardy during cold months; (b) males from Sicily during all of the months; and (c) males from Lazio during warm months[8].

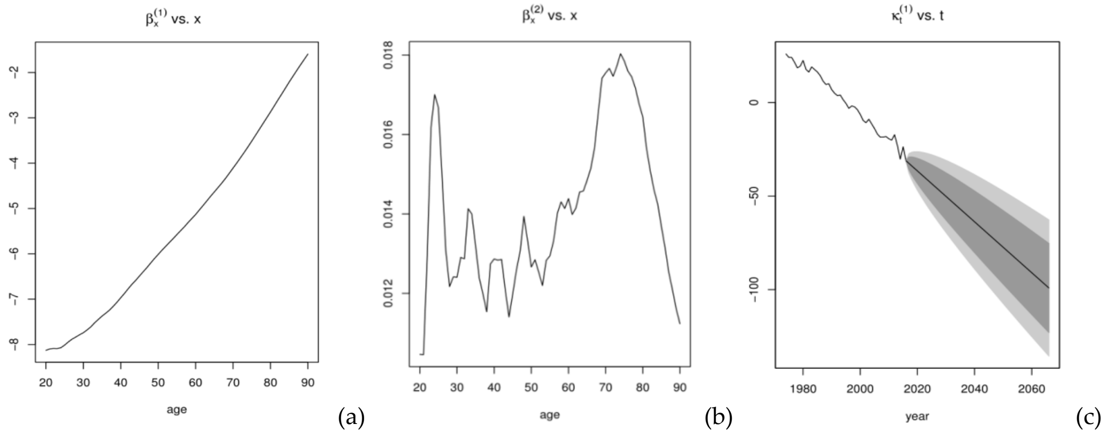

Fig. 3.3 – LC parameters for females from Lazio during cold months (with 50 years of forecast for $\boldsymbol{k_t^1}$)[9].

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[8] The diagonal lines heighten the presence of a cohort effect.

[9] The LC model parameter plots emerge as expected, considering the meaning of each parameter. Basically, the $\boldsymbol{b_x^1}$shape is almost the same for all similar populations. Although the $\boldsymbol{b_x^2}$ shape slightly differs when the data is changed, it generally shows a quick decreasing trend for the last ages. The $\boldsymbol{k_t^1}$

3.3 Plat

The Plat (*2009*) model (hereafter referred as the P model) is an extension of the LC model which includes the cohort effect. The P model adapts well to all the age ranges and considers the experience of the younger ages. The model is:

$$\ln(m_{x,t}) = b_x^1 + k_t^1 + (\bar{x} - x)\, k_t^2 + (\bar{x} - x)^+\, k_t^3 + \gamma_{t-x} + \varepsilon_{x,t} \tag{15}$$

Where:

b_x^1 is equivalent to the LC one; it reflects the general mortality path over ages.

k_t^1, k_t^2 and k_t^3 describe changes of mortality for different calendar years for, respectively, ages, different age classes (reflecting the historical observation) and younger ages.

The k_t^1 factor is fitted with a non-stationary ARIMA process, while it assumes a stationary mean reverting process for k_t^2 and k_t^3 factors for non-biologically reasonable results of the mortality curve using a non-stationary ARIMA process.

γ_{t-x} represents the cohort effect. It is modelled as a trendless Mean Reversion process since it is not expected to change over years.

$\bar{x}$ is the mean of the ages into consideration and $(\bar{x} - x)^+ = \max(\bar{x} - x, 0)$.

The P model has parameter identifiability problems, which may be solved by using transformation factors to identify them. The setting constraints are:

$$\begin{cases} \sum_{c=c_0}^{c_1} \gamma_c = 0 \\ \sum_{c=c_0}^{c_1} c\gamma_c = 0 \\ \sum_t k_t^3 = 0 \end{cases} \tag{16}$$

Where:

$c = t - x$.

---

trend is strictly decreasing over time but the width of the forecast intervals differs substantially among samples.

c_0 and c_1 are the earliest and the latest year of birth used to fit the cohort effect. The first two constraints enable the γ_{t-x} process to only consider the cohort effect, without compensation for age or calendar year effects. The last constraint helps to normalise the factors.

For the fitting process, it may be assumed that the number of deaths is described by a Poisson $(E_{x,t}m_{x,t})$ distribution (14). Under this assumption, $b_x^1, k_t^1, k_t^2, k_t^3$ and γ_{t-x} factors may be estimated by using an iterative algorithm which maximises the log-likelihood function:

$$L\ (\psi, D, E) = \sum_{x,t} D_{x,t} ln\ [E_{x,t} m_{x,t}(\Phi)] - E_{x,t} m_{x,t}(\psi) - ln(D_{x,t}!) \qquad (17)$$

This procedure leads to k_t^1, k_t^2, k_t^3 and γ_{t-x} time series, then suitable ARIMA processes are fitted to forecast, as well as the previous models.

3.4 O'Hare and Li

The O'Hare and Li (*2012*) model (hereafter referred as the OL model) is a modification of the P model:

$$ln(m_{x,t}) = b_x^1 + k_t^1 + (\bar{x} - x)\, k_t^2 + ((\bar{x} - x)^+ + ([\bar{x} - x]^+)^2)\, k_t^3 + \gamma_{t-x} + \varepsilon_{x,t} \qquad (18)$$

Where k_t^3 multiplies another coefficient in order to capture the non-linear (quadratic) effects in the lower ages, when the variance is higher due to the smaller number of deaths and the specific causes of death for this age range. The constraints are the same as those exposed for the P model:

$$\begin{cases} \sum_{c=c_0}^{c_1} \gamma_c = 0 \\ \sum_{c=c_0}^{c_1} c\gamma_c = 0 \\ \sum_t k_t^3 = 0 \end{cases} \qquad (16)$$

In order to fit the model, it may be assumed that $D_{x,t} \sim Poisson\ (E_{x,t}\,\mu_{x,t})$, and the parameters are estimated by maximising the log-likelihood function (17).

In some countries, this model returns an improved fitting quality compared to the LC model and the P model because it may capture the non-linear behaviour of log mortality at younger ages for mature ages.

3.5 Other models

Although they will not be used to make comparisons with the temperature-related model, the Renshaw and Haberman (*2006*) and the CBD (*2006*) models are briefly introduced here, owing to their importance in the literature.

3.5.1 Renshaw and Haberman (*2006*)

The Renshaw and Haberman *(2006)* model is an Age-Period-Cohort (APC) version of the LC model; that is, it is another generalisation of it, built by adding a cohort factor $\boldsymbol{\gamma_{t-x}}$. The Renshaw and Haberman *(2006)* model revisits a previous model released in 2003:

$$\ln(\boldsymbol{m_{x,t}}) = \boldsymbol{b_x^1} + \boldsymbol{b_x^2 k_t^1} + \boldsymbol{b_x^3 \gamma_{\mathrm{t-x}}} + \boldsymbol{\varepsilon_{x,t}} \tag{19}$$

Where $\boldsymbol{b_x^3}$ describes, for every age, how the mortality reacts to changes in the cohort effect. One of the variants of the model includes a modification by setting $\boldsymbol{b_x^3}$=*1*, which solves some stability problems. Four constraints are suggested to obviate the identifiability problems:

$$\begin{cases} \sum_x \boldsymbol{b_x^2} = \mathbf{1} \\ \sum_t \boldsymbol{k_t^1} = \mathbf{0} \\ \sum_{x,t} \boldsymbol{\gamma_{t-x}} = \mathbf{0} \\ \sum_x \boldsymbol{b_x^3} = \mathbf{1} \end{cases} \tag{20}$$

The first two constraints are the same as those of the LC model. Even so, the authors have recently offered two alternative constraints to obtain a better model fitting:

$$\begin{cases} \boldsymbol{k_t^1} = \mathbf{0} \\ \boldsymbol{b_x^2} > \mathbf{0} \end{cases} \tag{21}$$

Under the assumption of independence between period and cohort effects, $\boldsymbol{k_t^1}$ and $\boldsymbol{\gamma_{t-x}}$ are developed as ARIMA processes and the mortality projections are found by forecasting these time series. This model presents a proper fitting, especially for populations featured with an evident cohort effect. Nevertheless, it might suffer a lack of robustness since the likelihood function might not have an absolute maximum and the optimisation process might lead to several local maxima with marked differences between them. Therefore, it may occur that a different set of parameters results when using a different age range or a different period, which is not a desirable feature.

3.5.2 CBD (*2006*)

The CBD model *(2006)* has been proposed by Andrew Cairns, David Blake and Kevin Dowd, three English university professors. By testing their model using English and Welsh males in the age range of 60 to 80 years, they noticed both that there was a trend in the year of birth of the fitted cohorts and that the natural logarithm of the mortality odds assumed a linear relationship over time:

$$\boldsymbol{ln\left(\frac{q_{x,t}}{p_{x,t}}\right) = k_t^1 + (\bar{x} - x)\, k_t^2 + \varepsilon_{x,t}} \tag{22}$$

Where $\boldsymbol{k_t^1}$ and $\boldsymbol{k_t^2}$ are two stochastic processes which form a bivariate random walk with drift, according to the authors' data, and drive the forecast. The model has neither a static age function nor a cohort effect. It has no identifiability problems, so constraints are not requested. The authors have subsequently presented some variants which, as well as the other models, need transformation factors in order to be individuated. Therefore, the parameters estimation for this model is performed by using the Ordinary Least Squares (OLS) method:

$$\boldsymbol{O}_t(\boldsymbol{k}) = \sum_{x=x_1}^{x=x_m}\left(\boldsymbol{ln}\left(\frac{\widehat{q_{x,t}}}{\widehat{p_{x,t}}}\right) - \boldsymbol{k}_t^1 + (\overline{\boldsymbol{x}} - \boldsymbol{x})\boldsymbol{k}_t^2\right)^2 \tag{23}$$

Table 3.1 gives an overall view of the stochastic mortality models proposed in this chapter.

Stochastic mortality models	Non-parametric age function	Parametric age function
Does not include cohort term	Lee and Carter *(1992)*	CBD *(2006)*
Includes cohort term	Renshaw and Haberman *(2006)*	Plat *(2009)* O'Hare and Li *(2012)*

Table 3.1 – A simple classification of the stochastic mortality models presented in this chapter.

3.5.3 Cairns et al. (*2008*) criteria

The researchers Cairns, Blake and Dowd have presented criteria (Cairns, A.J.G., Blake D. and Dowd, K., "*Modelling and Management of Mortality Risk: A Review", 2008*) against which a model may be assessed:

1) Mortality rates should be positive.
2) The model should be consistent with historical data.
3) Long-term dynamics under the model should be biologically reasonable.
4) Parameter estimates should be robust relative to the period of data and range of ages employed.

5) Model forecasts should be robust relative to the period of data and range of ages employed.
6) Forecast levels of uncertainty and central trajectories should be plausible and consistent with historical trends and variability in mortality data.
7) The model should be straightforward to implement using analytical methods or fast numerical algorithms.
8) The model should be relatively parsimonious.
9) It should be possible to use the model to generate sample paths and calculate prediction intervals.
10) The structure of the model should make it possible to incorporate parameter uncertainty in simulations.
11) At least for some countries, the model should incorporate a stochastic cohort effect.
12) The model should have a non-trivial correlation structure.

An additional criterion has been suggested by H.J. Plat (*Essays on valuation and risk management for insurers, 2011)* and is the applicability for a full age range. Table 3.2 summarises the criteria satisfaction for the stochastic mortality models proposed in this chapter.

Satistaction of criteria mortality models	Lee & Carter (1992)	Renshaw & Haberman (2006)	CBD (2006)	Plat (2009)	O'Hare & Li (2012)
1) Positive mortality rates	+	+	+	+	+
2) Consistency historical data	+/-	+	+	+	+
3) Long-term biological reasonableness	+	+	+	+	+
4) Robustness of parameters	+	-	+	+	+
5) Robustness of forecast	+	-	+	+	+
6) Forecast biological reasonable	+/-	+	+	+	+
7) Ease of implementation	+	+	+	+	+
8) Parsimony	+	+/-	+	+/-	+/-
9) Possibility generating sample paths	+	+	+	+	+
10) Allowance for parameter uncertainty	+	+	+	+	+
12) Incorporation cohort effects	-	+	-	+	+
12) Non-trivial correlation structure	-	+/-	+	+	+
13) Applicable for full age range	+/-	+/-	-	+	+

Notes: + = satisfied; +/- = partly satisfied; - = Not satisfied.

Table 3.2 – Satisfaction of criteria for the mortality models presented in this chapter (Plat H.J., *Essays on valuation and risk management for insurers (2011)*).

Chapter 4

Data from Italy

Not only do temperature and climatic changes, including extreme temperatures[10] such as heat waves (*Meehl et al., 2004*) and extreme cold or weather conditions such as rain or wind[11], directly affect human mortality but they also have an indirect effect, by modifying the surrounded environment where populations live. A clearer view may be obtained by thinking about the climate impact on flora and fauna or on the spread of infectious diseases (see, for instance, *IPCC (2014)* or *Wu et al., 2017*). In the literature, many other authors have studied the correlation between the climate factor and death (e.g. *McMichael et al., 2006; Patz et al., 2005; Gosling et al., 2009*).

In this thesis, the relationship between mortality rates and three indexes of temperature (average, highest and lowest) is investigated for three Italian regions: Lombardy, Lazio and Sicily. Temperature indexes are chosen as a proxy of a more general climatic index for two main reasons. Firstly, because, although it is challenging to capture all the significant factors involved in climatic changes, the temperature effects on human life are considered worldwide to be a reasonable approximation of all the climatic events which may alter mortality; secondly, because the time series of temperature indexes are reliable and easy to obtain for most countries around the world. For example, another interesting factor which might be taken into account is air humidity, which is strictly related to temperature. In fact, the more humid the air is, the more the temperature is felt by the population, especially the warmest temperatures. Nevertheless, sometimes air humidity time series are not

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[10] According to *Patz et al., 2005*, extreme temperatures will increase in relation to the average temperature, especially in the mid-latitudes (i.e., in Italy).

[11] See, for instance, *Easterling et al., 2000* for the Italian region of Trentino.

available, and its relationship with temperature does not have a single representation.

Italy's terrain is considerably variegated. Not only does it differ among its regions in terms of temperature values, but also in economic and cultural features and in people's lifestyles. In this paper, three Italian regions are analysed: Lombardy, Lazio and Sicily. These regions reliably represent the three zones into which the country of Italy may be divided: these regions have been carefully chosen considering their positions (which is in the centre of the North, the Centre and the South), their population size (they are the most populated region in their area) and their temperature features (which is within the average of each zone). Data are presented in the following sections with the help of graphs, specifically mortality data (section 4.1), and temperature (section 4.2).

4.1 Mortality data

Mortality rates have continuously and quickly decreased over the last two centuries for all ages, although they have shown different results, with even better improvement in the last fifty years. People now live longer, primarily thanks to science and medical evolution which enables humans to reach, in many developed countries, a life expectancy at birth above 80 years old. Figure 4.1 shows Italian data over the last forty years for the male population up to 100 years old, and it shows that the decrease is remarkable for every age.

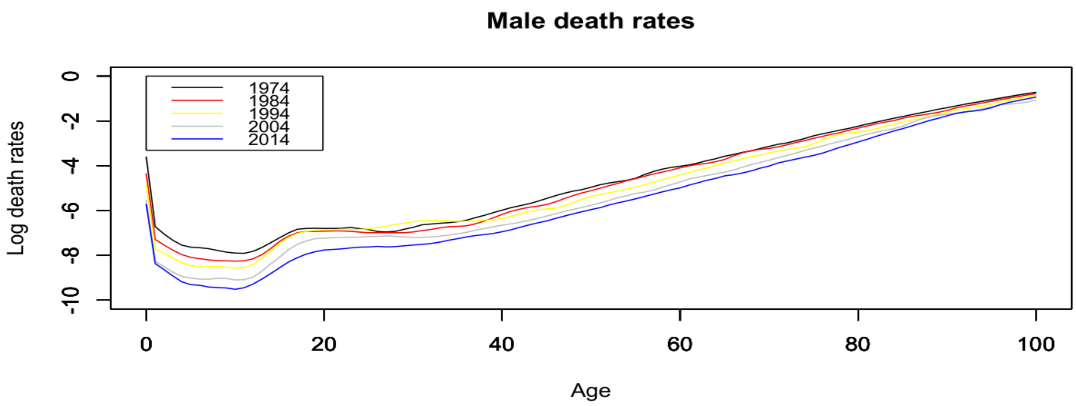

Fig. 4.1 – Italian male death rates, from 1974 to 2014.

To better understand the LC model, it may be considered that its terms idealistically represent some features of this graph: the general shape of the mortality rates over ages is captured by the coefficient $\boldsymbol{b_x^1}$, while the decrease over the years from 1974 to 2014 is represented by the factor $\boldsymbol{k_t^1}$. Lastly, the differences of that decrease for each age are modelled by $\boldsymbol{b_x^2}$. It is also interesting to focus solely on a 50-year-old male as an example of examining the decreasing central mortality rate trend over these years (Fig. 4.2).

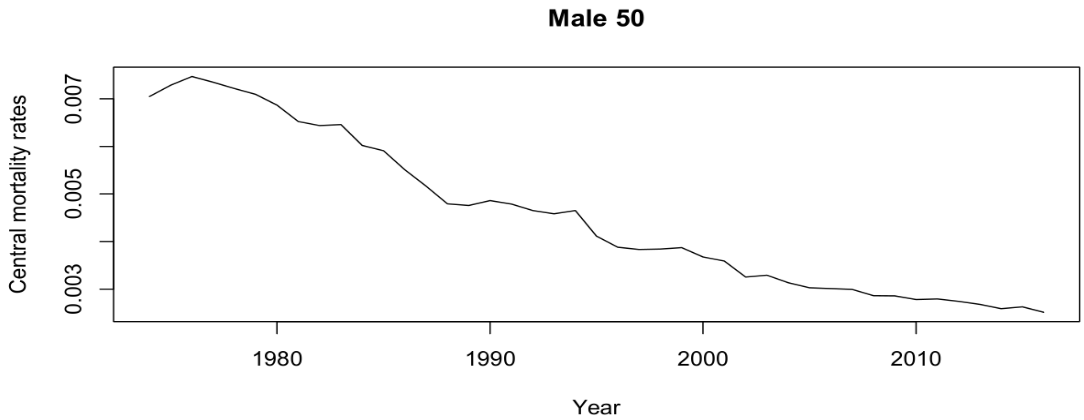

Fig. 4.2 – The central mortality rates path for a 50-year-old Italian male, over the period of 1974-2016.

Annual resident population data on the 1st of January and death probabilities for each year between 1974 and 2016, divided by gender, age and region, are provided by the Italian National Institute of Statistics (ISTAT). In order to obtain monthly data, the resident population on the 1st of every month is calculated by using linear interpolation, which is considered the best approximation procedure for this analysis. Death probabilities, on the other hand, are adjusted through a monthly trend factor, which is calculated by observing the monthly deaths data, available for the period November 2011 – July 2016[12], and by assuming a constant trend over the previous years. Finally, a monthly average over the warm months, over the cold months and over all of the months is calculated in order to obtain seasonal mortality data. The age

[12] The record of October 2011 is available, too, but it has been eliminated during the data analysis process, since it is considered unreliable.

range is limited from 20 to 85, and a constant force of mortality is assumed for each sub-period so that the central mortality rates $\boldsymbol{m_{x,t}}$ may be calculated as in the formula:

$$\boldsymbol{m_{x,t} = -\ln(1-q_{x,t})} \tag{24}$$

Where $\boldsymbol{q_{x,t}}$ is the probability of death.
Furthermore, it is used the relationship which links exposed to risk $\boldsymbol{E_{x,t}}$, central mortality rate $\boldsymbol{m_{x,t}}$ and the number of deaths $\boldsymbol{D_{x,t}}$:

$$\boldsymbol{D_{x,t} = E_{x,t}{}^{*}\, m_{x,t}} \tag{25}$$

In order to show the mortality differences for the critical variables, four plots are displayed on a semi-logarithm scale (Fig. 4.3 (a) and (b); Fig 4.4 (a) and (b)) with a thirty-year projection based on an LC model with Poisson distribution of deaths (see section 3.2.3) for the first three. The age and gender variables are respectively scanned in Figure 4.3 (a) for a male from Lazio and in Figure 4.3 (b) for a 75-year-old Sicilian person.

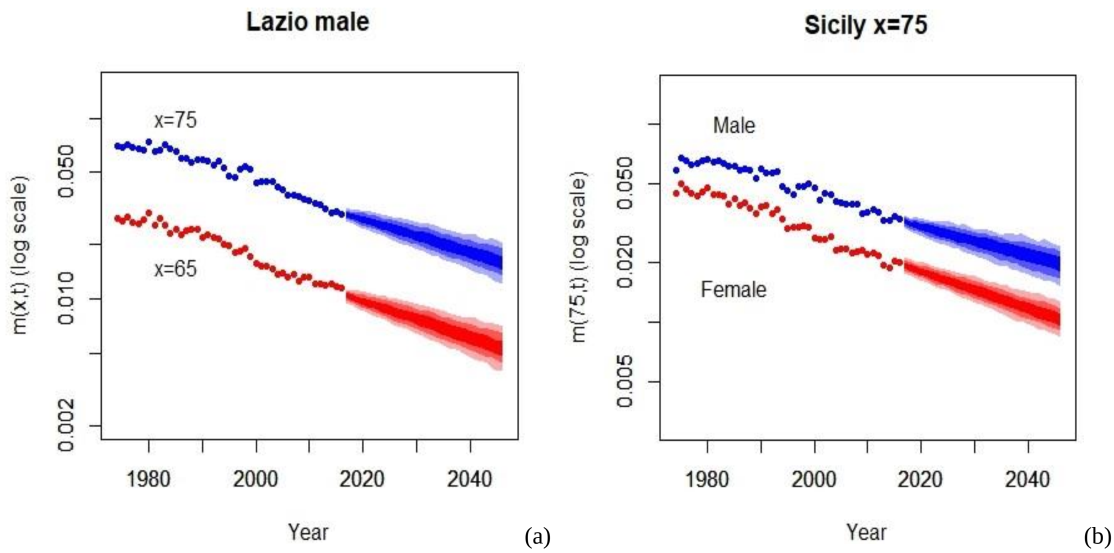

Fig. 4.3 – Comparisons of the mortality among the variables “age” (a) and “gender” (b).

In addition, the difference between the rates for warm months and cold months (see the following section for more details) for an 80-year-old female from Lombardy is displayed in Figure 4.4 (a). Lastly, Figure 4.4 (b) heightens the importance of the regional variable.

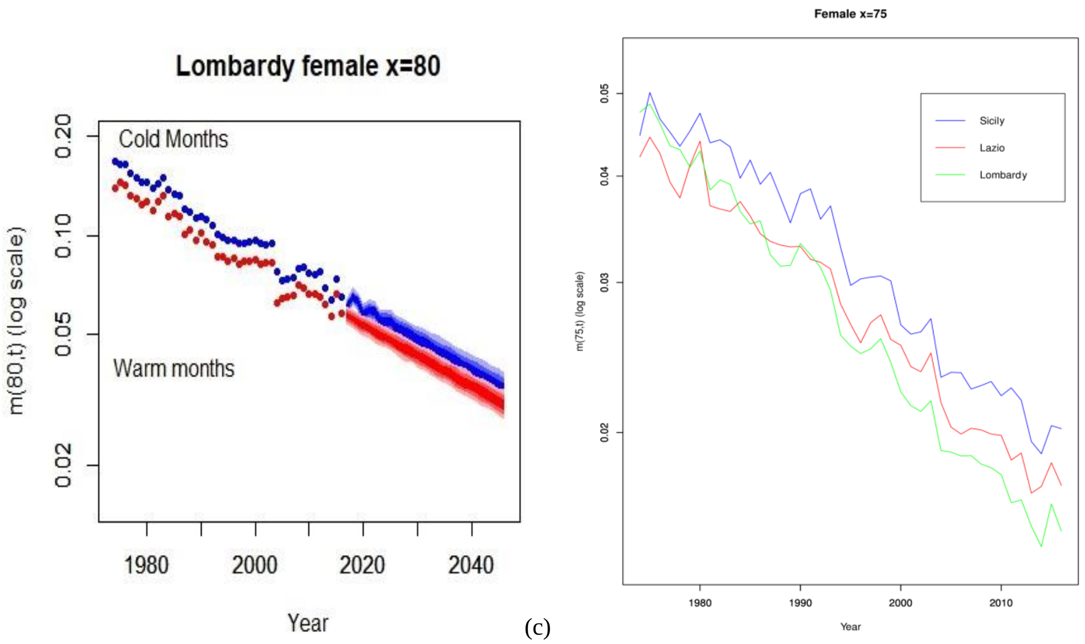

Fig. 4.4 – Comparisons of the mortality among the variables “period of the year” (a) and “region” (b).

4.2 Temperature data

The mean surface temperature in Italy has increased by about 2.5°C during the last century. After analysing whether the mean monthly temperature records are either below or above the average annual temperature, the calendar years are divided into two equal parts. The first part is referred to as the cold months, from November to April, the second part reflects the warm months, from May to October. Of course, the annual data have not been left out, and they will continue to be studied.

Temperature data are recorded for the same period by the weather station of “Milano Linate” for Lombardy, by the weather station “Roma Collegio Romano” for Lazio and by the “Palermo Punta Raisi” weather station for Sicily. Milan, Rome and Palermo are the chief cities of the regions of Lombardy, Lazio and Sicily, respectively; they are situated in the middle of their region, and they include a majority of their population (moreover, they represent three of the top five Italian cities in terms of population size). Hence, for this mortality study, their temperature series may be considered a proper proxy for their region.

Temperatures in Northern Italy differ from those of Central or Southern Italy. The main differences are not observed in the average temperature[13] or in the highest one[14], but in the lowest temperature. This is very interesting, since the results of Seclecka, Pantelous and O'Hare showed that mortality rates are more correlated with the cold months, when the coldest temperatures occur[15]. This result advances the idea that a difference of the model validity among regions may be discovered. By focusing on the cold months, the arithmetic mean of the lowest temperatures over the period 1974-2016 records 2.83°C for Lombardy, 5.62°C for Lazio and 11.37°C for Sicily. This difference is notable considering that during the warm months the same value is 15.07°C for Lombardy, which is definitely closer to 11.37°C than 2.83°C. In addition, it is important to remember that these are arithmetic means over 43 years and the records are more meaningful by observing, for example, the absolute minimum monthly lowest temperature, which is -5.6°C for Lombardy, -0.6°C for Lazio and an astounding 7.3°C for Sicily[16]. Figure 4.5 (a), (b), (c), (d) shows the arithmetic mean of the highest, average and lowest temperature records for the three regions, combined in annual and seasonal data over the period 1973-2017 with an additional trend line for the next ten years. Although the period is not very long, an increase is overt in every trend.

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[13] For instance, the annual mean of the average temperature is 13.41°C for Lombardy, 17.02°C for Lazio and 18.77°C for Sicily.

[14] For instance, the arithmetic mean of the highest temperature is 18.31°C for Lombardy, 21.24 for Lazio and 21.69°C for Sicily when the entire year is considered, while it is 25.47°C for Lombardy, 27.37°C for Lazio and 26.36°C for Sicily when only the period of the warm months is considered.

[15] In fact, they did not consider the lowest temperature data.

[16] Respectively, on February 1991, February 2003 and January 2014.

These analyses accept the hypothesis that a negative correlation may occur between temperature and mortality rates and suggest proceeding to study the correlation between these two variables.

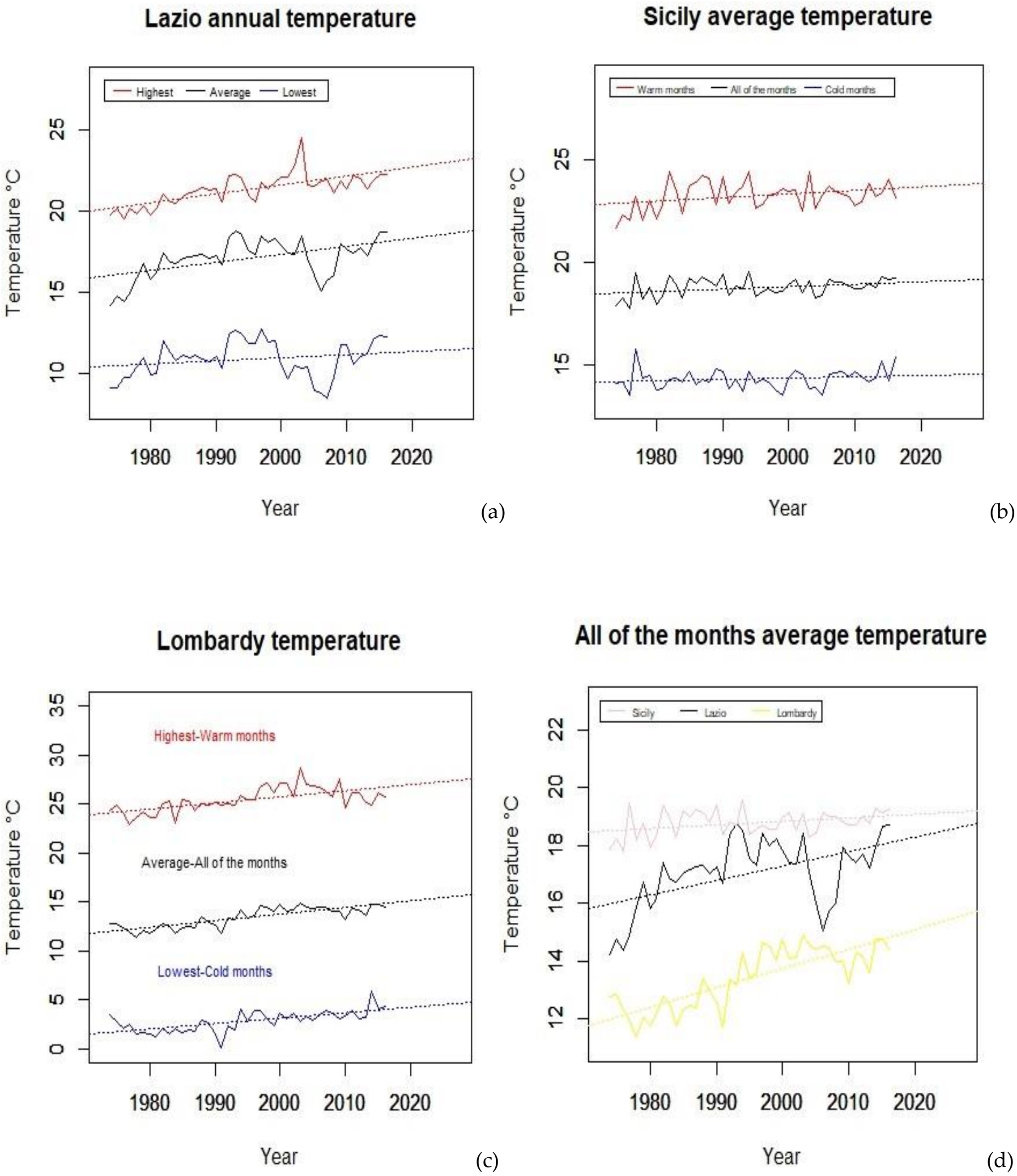

Fig. 4.5 – Analyses of selected temperature series:
(a) Lazio, all of the months. (b) Sicily, average temperature.
(c) Lombardy. (d) Average temperature, all of the months.

Chapter 5

Trends and relationships among data

In this chapter, the study of the Lee & Carter mortality index $\boldsymbol{k_t^1}$ and changes in temperature fluctuations is deepened, and a statistical relationship between them is searched[17]. In addition, a correlation coefficient between the central mortality rate $\boldsymbol{m_{x,t}}$ and temperature is sought, and its robustness is checked. The results strengthen the reliability of building a temperature-related factor model.

5.1 Trends in the LC mortality index and the temperature fluctuations

Following a similar pattern of the works of Hanewald *(2011)*, Niu and Melberg *(2014)*[18] and Seklecka, Pantelous and O'Hare *(2017)*, unit root tests are performed on both the series to better analyse their behaviour. In so doing, it must be borne in mind that the variables do not have many observations since it is being studied over a forty-three year period. Thus, the subsequent interpretation and trustworthiness of the outputs must take that fact into account. The variables under consideration are the Lee and Carter mortality index $\boldsymbol{k_t^1}$[19] and the logarithm of the highest, average and lowest temperature fluctuations for warm months, cold months and all months.

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[17] It is useful to remind the reader that a statistical relationship does not automatically imply a cause-effect relationship.

[18] Particularly, they have studied the relationship between the $\boldsymbol{k_t^1}$ of the Lee & Carter model and several economics indices.

[19] The Lee & Carter model parameters have been estimated by using the *Brouhns et al. (2002)* Poisson distribution variant.

Firstly, the Phillips-Perron (PP) test (Perron *(1998)*) is performed. The PP test is a generalisation of the augmented Dickey-Fuller (ADF) test and tests the null hypothesis of non-stationarity of the time series (i.e. that they have a unit root) against a stationary alternative. The PP test may be applied with the inclusion of a constant, a constant and a linear trend, or neither. By observing the plots of the mortality index $\boldsymbol{k_t^1}$ from 1974 to 2016, it is clear that it exhibits pronounced downward trend. In the same period, all the temperature series seem to display lightly raising fluctuations, however such a trend is not evident and is not really logical (see Fig. 3.3 (c) and Fig. 4.5). Therefore, the test is applied including both a constant and a linear trend, which is also the most general setting with which to proceed, for the $\boldsymbol{k_t^1}$, but without a linear trend for the temperature series. Results are shown in Table 5.1.

Secondly, the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test is performed, with the same assumption used for the linear trends of the time series as discussed for the PP. The KPSS, on the contrary, tests the null hypothesis of stationarity of a time series around a deterministic trend, against the alternative of a unit root. Through this test and considering the results of the PP test and the plots of the time series, the absence of a unit root may be the proof of a trend stationary series. Results are shown in Table 5.2.

The PP findings for males and females and for temperature series are different. Considering the latent factor $\boldsymbol{k_t^1}$**,** the null hypothesis of non-stationarity is rejected at the 5% significance level for females from each region without exception, while for males it is rejected for 5 of the 9 mortality indexes (but cannot be rejected for 2 of these at the 10% significance level). Therefore, according to the results of the PP test, and since the linear time trend is meaningful in these series, they are found to be trend stationary. With respect to the logarithm of the temperature fluctuations, once again the results differ among regions. The null hypothesis is rejected in every case for Sicily at the 5% significance level, but in the case of Lazio, there is no evidence to reject it for the average and the lowest temperatures of warm months and all months at the same level, while for the highest temperature it is always rejected. Considering

Lombardy, the null hypothesis is only rejected at this level for the highest temperature of the warm months and the lowest temperature of the cold months. In total, the null hypothesis cannot be rejected for 11 cases of the 27 temperature indexes; hence, it is quite difficult to discover a general behaviour.

Scanning the outputs of the KPSS test, with respect to the mortality index $\boldsymbol{k_t^1}$, the null hypothesis is rejected for males from each region at the 5% significance level without exception, but it cannot be rejected for females from Lazio nor from Lombardy (only for cold months). Concerning the temperature, at the same level, the null hypothesis is always rejected for Lombardy, but only for the annual highest temperature for Sicily. Finally, for Lazio, the situation lies in the middle: the null is always rejected for the highest temperature but never for the lowest. In general, the test does not give homogeneous results. In fact, as is the case with the PP test, the null hypothesis cannot be rejected for 11 of the 27 series at the 5% significance level.

Tables 3.1 and 3.2 show various results among regions because distinct populations behave differently and the temperature changes substantially in different territories. Furthermore, comparing the PP test with the KPSS test, the findings may sometimes seem contradictory: it is worth remembering that these tests perform asymptotically and that in a finite sample is very difficult to distinguish between a trend-stationary and a difference-stationary behaviour. The first one may be made stationary by removing the deterministic trend, while the second one, after differencing. By applying the wrong transformation, there is a risk of incurring grave consequences[20]. Therefore, drawing general conclusions is complicated, and it is better to stop the analysis of the series at levels. In the next section, similarly to what other studies have done for macroeconomic indexes (see, for example, *J.A. Tapia Granados (2008), (2011))*, the

[20] Hence, with these data, it is not very sensible to proceed by studying the co-integration of the series (either with the Johansen test or Engle Granger's test), as Seklecka, Pantelous and O'Hare have done. In fact, two time series are co-integrated when they are integrated and a linear combination of them is stationary; thus, it may be interpreted as a long-run relationship between the variables. Moreover, it would be sensible to study co-integration between two series which graphically behave "similarly".

statistical association between the mortality index $\boldsymbol{k_t^1}$ and temperature will be further examined by calculating the Pearson's correlation coefficient, in order to attempt to give a climatic interpretation to the latent factor $\boldsymbol{k_t^1}$.

Phillip-Perron (PP)	Gender		All of the Months	Cold Months	Warm Months
Lazio	Male	k^1_t	-3,4710*	-2,1338	-3,4616*
	Female	k^1_t	-4,9817***	-4,7140***	-5,4862***
	Ln of Temperature	high	-3.6278***	-3.4536**	-3.4239**
		avg	-2.4106	-3.154**	-2.6155*
		low	-2.6877*	-3.9649***	-2.6802*
Lombardy	Male	k^1_t	-4,8150***	-3,8985**	-4,7918***
	Female	k^1_t	-4,9467***	-5,9811***	-6,0355***
	Ln of Temperature	high	-1.8368	-2.1879	-3.1182**
		avg	-1.5469	-2.7483*	-2.2334
		low	-2.2686	-5.2758***	-2.384
Sicily	Male	k^1_t	-4,6331***	-3,1597	-4,4753***
	Female	k^1_t	-5,4315***	-3,9259**	-6,1121***
	Ln of Temperature	high	-3.1683**	-4.9266***	-3.5448**
		avg	-6.1941***	-6.8522***	-5.8765***
		low	-5.1883***	-4.7476***	-6.1685***

Notes: *p<0.1, **p<0.05, ***p<0.001
Critical values for k^1_t are: -4.1896 for p<0.01, -3.5189 for p<0.05 and -3.1898 for p<0.1

Critical values for temperature are: -3.5849 for p<0.01, -2.9286 for p<0.05 and -2.6020 for p<0,1

Table 5.1 – Phillip-Perron test of the $\boldsymbol{k_t^1}$ and the natural logarithm of the temperature series.[21]

Kwiatowski et al. (KPSS)	Gender		All of the Months	Cold Months	Warm Months
Lazio	Male	k^1_t	0.2284***	0.2251***	0.2179***
	Female	k^1_t	0,1115	0,1086	0,1069
	Ln of Temperature	high	0.8968***	0.9132***	0.7003**
		avg	0.4881**	0.5181**	0.4256*
		low	0.1783	0.1032	0.1821
Lombardy	Male	k^1_t	0.2380***	0.2469***	0.2382***
	Female	k^1_t	0.1605**	0.1303*	0.1520**
	Ln of Temperature	high	0.7852***	0.6977**	0.7881***
		avg	0.9649***	0.8228***	0.9731***
		low	1.0134***	0.4807**	1.0472***
Sicily	Male	k^1_t	0.2560***	0.2617***	0.2600***
	Female	k^1_t	0.1621**	0.1702**	0.1640**
	Ln of Temperature	high	0.4982**	0.4368*	0.4424*
		avg	0.4374*	0.2546	0.3783*
		low	0.2910	0.3530*	0.2002

Notes: *p<0.1, **p<0.05, ***p<0.001
Critical values for k^1_t are: 0.216 for p<0.01, 0.146 for p<0.05 and 0.119 for p<0.1

Critical values for temperature are: 0.739 for p<0.01, 0.463 for p<0.05 and 0.347 for p<0,1

Table 5.2 – KPSS test of the $\boldsymbol{k_t^1}$ and the natural logarithm of the temperature series.[19]

[21] To calculate the results of the tests, the ur.pp and the ur.kpss functions, from the R package "urca", have been used (for more details, see: https://cran.r-project.org/web/packages/urca/).

5.2 Correlation coefficients between the LC mortality index and temperature fluctuations

According to the Seklecka, Pantelous and O'Hare *(2017)* article "Mortality effect of temperature changes in the United Kingdom", the relationship between the mortality index $\boldsymbol{k_t^1}$ and the temperature fluctuations is investigated by testing the association between paired samples using the Pearson's correlation coefficient; trying to give a climatic interpretation to the Lee & Carter latent factor $\boldsymbol{k_t^1}$. Moreover, to check the robustness of the results, the rank correlations Kendall's $\boldsymbol{\tau}$ and Spearman's $\boldsymbol{\varrho}$ are also calculated. The Pearson product-moment correlation coefficient (PPMCC) is a parametric measure of the linear association between two numeric variables. Kendall's rank correlation is a non-parametric measure of the association of x-y pairs based on concordance or discordance. Spearman's rank correlation is another non-parametric measure of the monotonic association between two numeric variables.

The outcomes, for each gender and all three highest, average and lowest temperatures, are shown in Table 5.3 for the Pearson coefficient, Table 5.4 for the Kendall coefficient and Table 5.5 for the Spearman coefficient. To simplify the reading, the greater results among temperature series of warm months, cold months and all of the months are highlighted in bold. The purpose is to attempt to draw a conclusion by comparing them to the ones displayed in the next section. The t-test, with the null hypothesis that a particular coefficient is equal to zero at a significance level of 5%, is also performed for the Pearson correlation coefficients. All correlations are negative and are occasionally very strong. Furthermore, by choosing any variable among the region, gender, temperature series or period of the year, the three tests display the same ranking, with similar numerical results most of the time[22].

By analysing the Pearson's coefficient regionally, Lazio shows the strongest correlations for the highest temperature, with a vast difference from both the

[22] The Kendall rank correlation coefficient usually gives lower results than the other two measures.

average and the lowest temperatures, for each gender and considering cold months, all of the months or, more weakly, the warm months. In fact, for both males and females, and for all three periods – warm months, cold months and all of the months – the null hypothesis of no correlation only for the lowest temperature is not rejected.

With respect to Lombardy, on the other hand, every correlation is unquestionably significant, especially for the average and lowest temperature, and it is difficult to state which one of them is more correlated between them. By considering the entire year, the most reliable correlation manifests itself for the lowest temperature series, although the difference between the Pearson correlation coefficient of the average temperature series for females is negligible (-0.8298 versus -0.8263) and very small for males (-0.8375 versus -0.8194)[23]. On the contrary, for the cold months, the average temperature series gives stronger correlations for each gender with a broader difference with the lowest than previously. Lastly, with respect to the warm months, Kendall's and Pearson's coefficients suggest taking the lowest temperature series irrespectively from the gender, but the average is preferable[24] when using the Spearman coefficient.

Sicily presents the smallest correlations among the regions. The Pearson coefficient exceeds the absolute values of 0.4 only for the association between female mortality and the annual highest temperature. For this region, only 4 correlations over 18 may reject the null hypothesis (22%). To a certain extent, the internal differences among highest, average and lowest temperature are similar, but more moderate, than those of Lazio.

On the whole, 34 of the 54 (63%)[25] correlations are significant, which evidently exceeds the 5% which may be expected in any random data set.

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[23] The other measures mark larger differences. The Kendall correlation coefficient provides the values of -0.6441 versus -0.5776 for females and -0.6530 versus -0.5865 for males, for lowest and average temperatures, respectively; while those of the Spearman coefficient are -0.8425 versus -0.7874 for females and -0.8487 versus 0.7960 for males.

[24] Even in this case, the difference is negligible: -0.7969 versus -0.7925 for females and -0.8062 versus -0.8022 for males, for average and lowest temperatures, respectively.

[25] Specifically: 67% for Lazio, 100% for Lombardy and 22% for Sicily.

Pearson	Gender	Temperature	All of the months	Cold Months	Warm Months
Lazio	Male	high	**-0,5948***	**-0,6727***	**-0,5076***
		avg	-0,4355*	-0,4979*	-0,3420*
		low	-0,1883	-0,1388	-0,1326
	Female	high	**-0,6073***	**-0,6806***	**-0,5259***
		avg	-0,4724*	-0,5344*	-0,3730*
		low	-0,2193	-0,1694	-0,1572
Lombardy	Male	high	-0,6845*	-0,6258*	-0,6228*
		avg	-0,8194*	**-0,7181***	-0,7706*
		low	**-0,8375***	-0,6533*	**-0,8074***
	Female	high	-0,7067*	-0,6401*	-0,6450*
		avg	**-0,8263***	**-0,7122***	-0,7865*
		low	**-0,8298***	-0,6250*	**-0,8212***
Sicily	Male	high	**-0,3608***	**-0,3797***	**-0,2420**
		avg	-0,2719	-0,2481	-0,1785
		low	-0,2562	-0,2222	-0,2141
	Female	high	**-0,4119***	**-0,3976***	**-0,2977**
		avg	-0,2855	-0,2236	-0,2057
		low	-0,2753	-0,2563	-0,2206

Notes= *p-value<0.05

Table 5.3 – Pearson's correlation coefficients between temperature series and the LC mortality index $\boldsymbol{k_t^1}$.

Kendall	Gender	Temperature	All of the months	Cold Months	Warm Months
Lazio	Male	high	**-0,5022**	**-0,5635**	**-0,3328**
		avg	-0,3714	-0,3817	-0,2359
		low	-0,1773	-0,1543	-0,1517
	Female	high	**-0,5111**	**-0,5480**	**-0,3306**
		avg	-0,3758	-0,3789	-0,2381
		low	-0,1906	-0,1565	-0,1539
Lombardy	Male	high	-0,4839	-0,4321	-0,4640
		avg	-0,5865	**-0,4859**	-0,5939
		low	**-0,6530**	-0,4565	**-0,6019**
	Female	high	-0,4751	-0,4277	-0,4485
		avg	-0,5776	**-0,4859**	-0,5739
		low	**-0,6441**	-0,4476	**-0,5908**
Sicily	Male	high	**-0,2007**	**-0,2833**	**-0,1644**
		avg	-0,1865	-0,1793	-0,1120
		low	-0,1210	-0,1371	-0,0676
	Female	high	**-0,2007**	**-0,2789**	**-0,1732**
		avg	-0,1954	-0,1837	-0,1253
		low	-0,0988	-0,1198	-0,0543

Table 5.4 – Kendall's correlation coefficients between temperature series and the LC mortality index $\boldsymbol{k_t^1}$.

Spearman	Gender	Temperature	All of the months	Cold Months	Warm Months
Lazio	Male	high	**0,6745**	**-0,7626**	**-0,4964**
		avg	-0,5139	-0,5430	-0,3494
		low	-0,2563	-0,2199	-0,2214
	Female	high	**-0,6790**	**-0,7522**	**-0,4917**
		avg	-0,5137	-0,5350	-0,3432
		low	-0,2617	-0,2145	-0,2150
Lombardy	Male	high	-0,6851	-0,6316	-0,6685
		avg	-0,7960	**-0,7036**	**-0,8062**
		low	**-0,8487**	-0,6482	-0,8022
	Female	high	-0,6767	-0,6275	-0,6586
		avg	-0,7874	**-0,7018**	**-0,7969**
		low	**-0,8425**	-0,6395	-0,7925
Sicily	Male	high	**-0,3028**	**-0,4008**	**-0,2793**
		avg	-0,2546	-0,2719	-0,1875
		low	-0,2155	-0,2035	-0,1441
	Female	high	**-0,3054**	**-0,3954**	**-0,2858**
		avg	-0,2650	-0,2789	-0,2013
		low	-0,1862	-0,1829	-0,1152

Table 5.5 – Spearman's correlation coefficients between temperature series and the LC mortality index $\boldsymbol{k_t^1}$.

5.3 Correlation coefficients between the mortality rates (by ages) and temperature fluctuations

Because the results in the previous paragraph are not strong enough to be able to provide a climatic interpretation for the latent factor $\boldsymbol{k_t^1}$, the Pearson's correlation coefficient between the temperature series and the central mortality rates $\boldsymbol{m_{x,t}}$ for each dataset is also analysed. These correlations will be directly implemented in the model structure (see Chapter 6). The results are presented in Figure 5.1 for Lazio, Figure 5.2 for Lombardy and Figure 5.3 for Sicily. Only the ages from approximately 40-45 years and older are discussed, because of the model structure (see section 6.1) and because the correlations are stable and strong in this range.

Lazio shows the most robust correlations for the highest temperature, in which the correlations never fall short of the absolute value of 0.5, with a marked difference from the average temperature (they never reach -0.6, but they are still interesting) and a primary difference with the lowest temperature (they remain in the range $[-0.2, 0.1]$ and do not exhibit a remarkable correlation). Therefore, it seems the highest temperature will give the best fitting and forecasting results[26].

The extraordinary correlation results for Lombardy are similar to the ones seen in the previous section. They almost always present an absolute value above 0.7 for both the warm months and all of the months, and a slightly lower value for the cold months, where it is assessed around the range $[-0.75, -0.5]$. It is not an easy task to choose between the average and the lowest temperature series for cold months where they lay on top of each other, while for both the warm months and all of the months, the average temperature results are continuously more correlated than the lowest temperatures.

Not only does Sicily present the lowest absolute values among all three regions, as was expected considering the climatic features of this region (see section 4.2) and the temperature-related causes of death previously discussed (see Chapter 4 and Introduction), but, contrary to all the other findings, the correlation between $m_{x,t}$ and the lowest temperature is strictly positive[27]. The highest temperature always results as more correlated to the central mortality rate than the others, in absolute values. In any case, the outputs are generally less high, but still interesting, than those found for Lazio and, above all, for Lombardy: they assess around the range $[-0.4, -0.6]$ with no substantial differences among different datasets. These findings primarily derive from the mild climate of the region. Thus, for this region, a lower improvement in fitting and forecasting quality is expected using the proposed model instead of the LC, P or OL models. Finally, Sicily also presents a different correlation shape over the ages compared with the other regions: in most cases it does not display a dump around the age of 30 years but remains rather linear.

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[26] This assumption is supported by the findings of the article by Seklecka, Pantelous and O'Hare.

[27] This feature, however, does not impact the model because the square of it is implemented. For more details, see the following chapter.

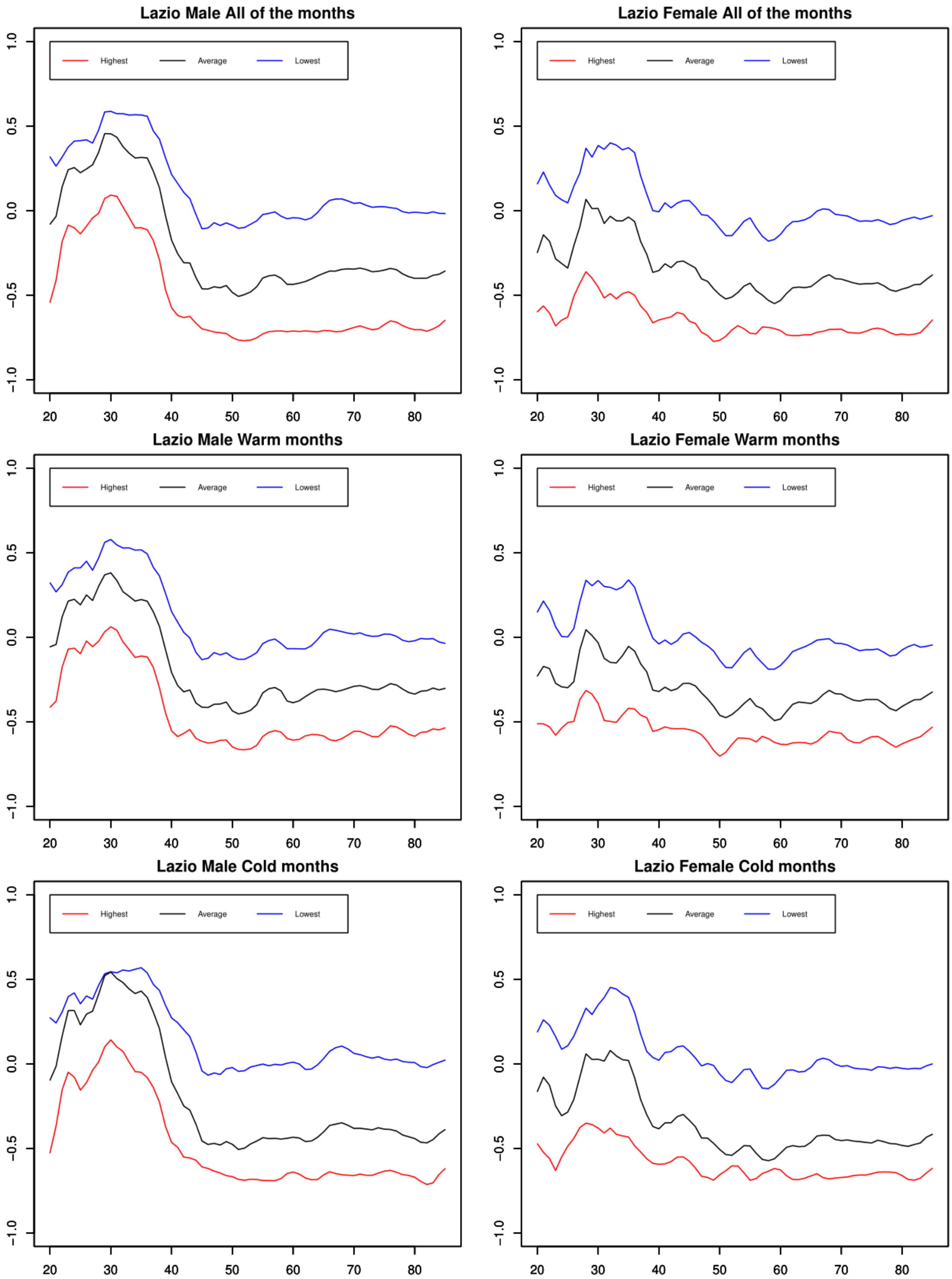

Fig.5.1 – Pearson's correlation coefficient between temperature series and the central mortality rates $\boldsymbol{m_{x,t}}$ for Lazio.

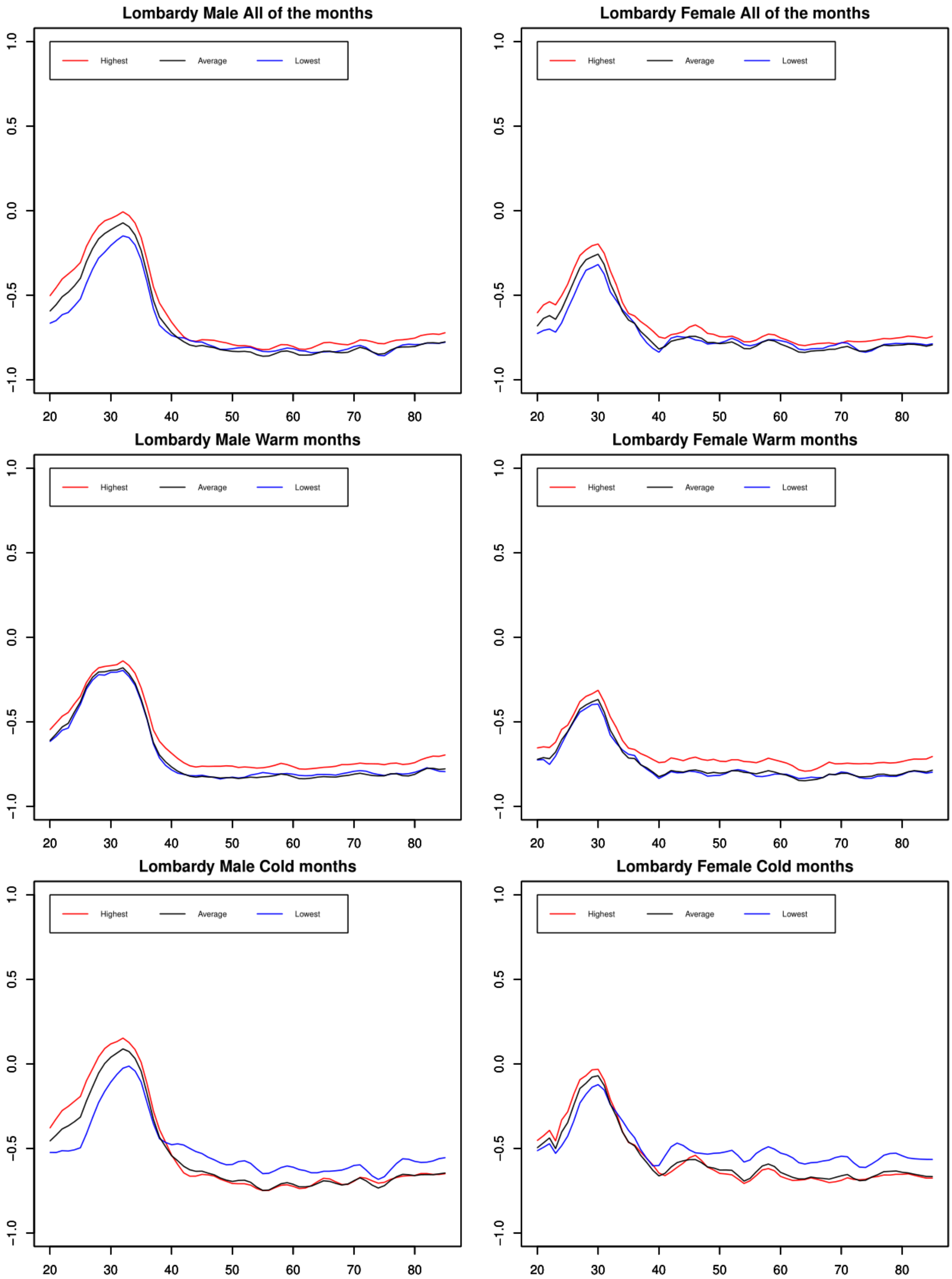

Fig. 5.2 – Pearson's correlation coefficient between temperature series and the central mortality rates $m_{x,t}$ for Lombardy.

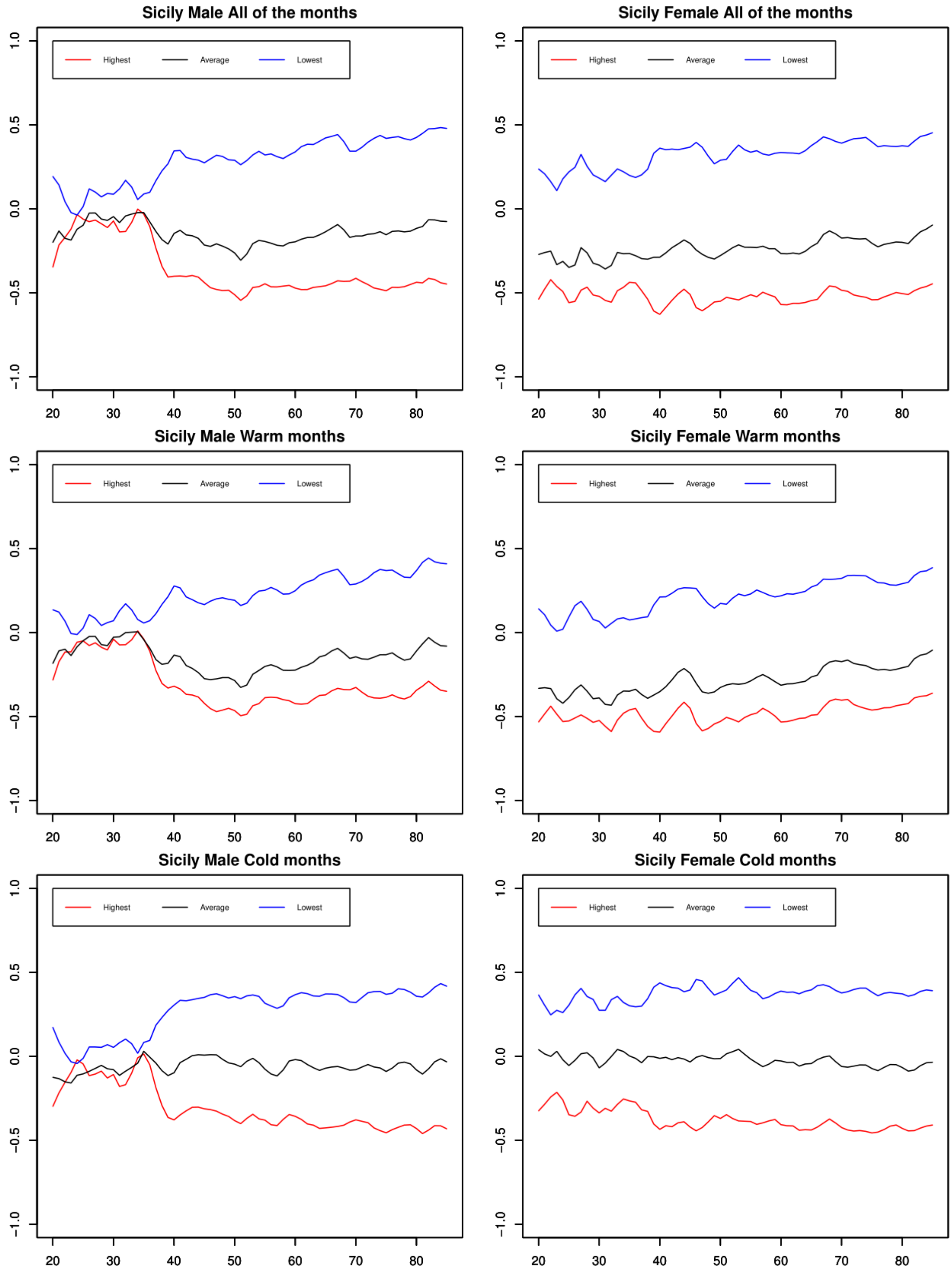

Fig 5.3 – Pearson's correlation coefficient between temperature series and the central mortality rates $m_{x,t}$ for Sicily.

5.3.1 Robustness check

The findings substantially support the assumptions that each region should be studied separately and that the highest and lowest temperature series may be meaningful. Moreover, the often-significant statistic relationships found between temperature and both $\boldsymbol{k_t^1}$ and $\boldsymbol{m_{x,t}}$ validate the study of the model. Before doing so, however, it is necessary to check the robustness of the Pearson coefficients for $\boldsymbol{m_{x,t}}$ and the temperature fluctuations since their direct implementation in the model. In the interest of not unnecessarily complicating this thesis, only the most significant series of the previous section are plotted below for each region. The other series are displayed in Appendix A[28]. Various periods are considered: from 1974 to 2016 (43 years), from 1974 to 2011 (38 years), from 1974 to 2006 (33 years) and from 1974 to 2001 (28 years) – each period for warm months, cold months and all months. All the considerations refer to ages above 40-45 years, according to the considerations given in section 5.3, which are also confirmed by Figures 5.4, 5.5 and 5.6, which show the results for Lazio, Lombardy and Sicily, respectively. With respect to Lazio, the analysis shows similar results for males and females. The plots indicate high robustness for all the periods; for the warm months, which are the least robust, the coefficient differences never reach 0.2. The correlations are high and stable. For Lombardy, the findings are as robust as those of Lazio, except for the 28-year period (from 1974 to 2001). In fact, the other correlations almost run together along the ages. Even for Sicily, the robustness is excellent for the cold months, while the results are weaker for the warm months and all of the months: the differences reach 0.35. In addition, the more years considered, the lower the correlation results. Lastly, the general shape of the graph of correlation for females does not possess the dump in the younger ages, as opposed to that which is shown in the other regions. In conclusion, the results of the robustness check are similar for males and for females, and the general shape of the

[28] Similar good results, although less robust, are generally displayed for Lazio, and for Lombardy.

correlation coefficient graph remains when the data period is changed. Therefore, the age-specific temperature-related factor is not very sensitive to the data range used during the parameter fitting process, and great robustness is observed. Hence, the model may be fitted with confidence in its reliability.

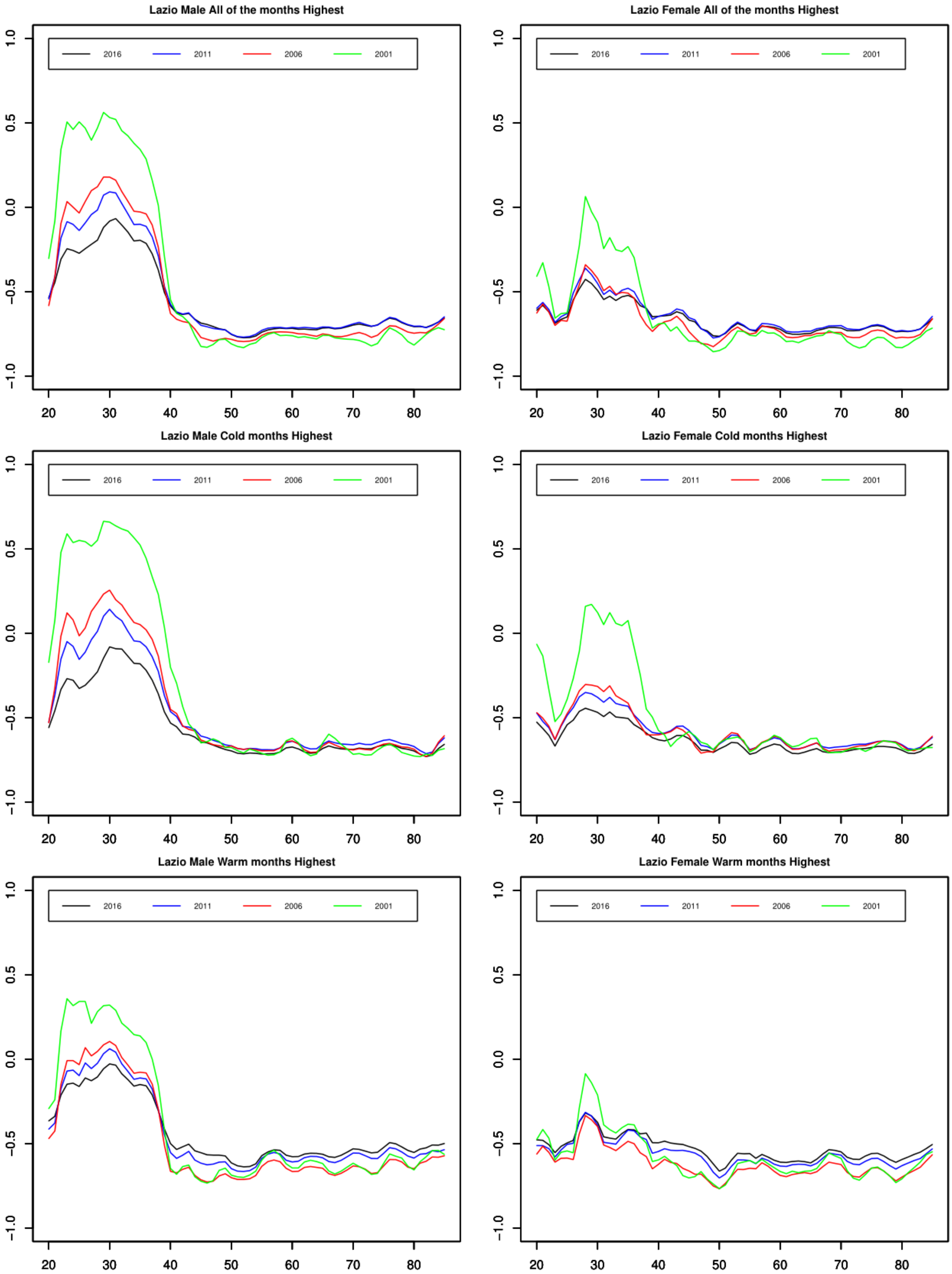

Fig 5.4 –Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and the highest temperature for various periods of time for Lazio, by gender, from 1974 to the reported year.

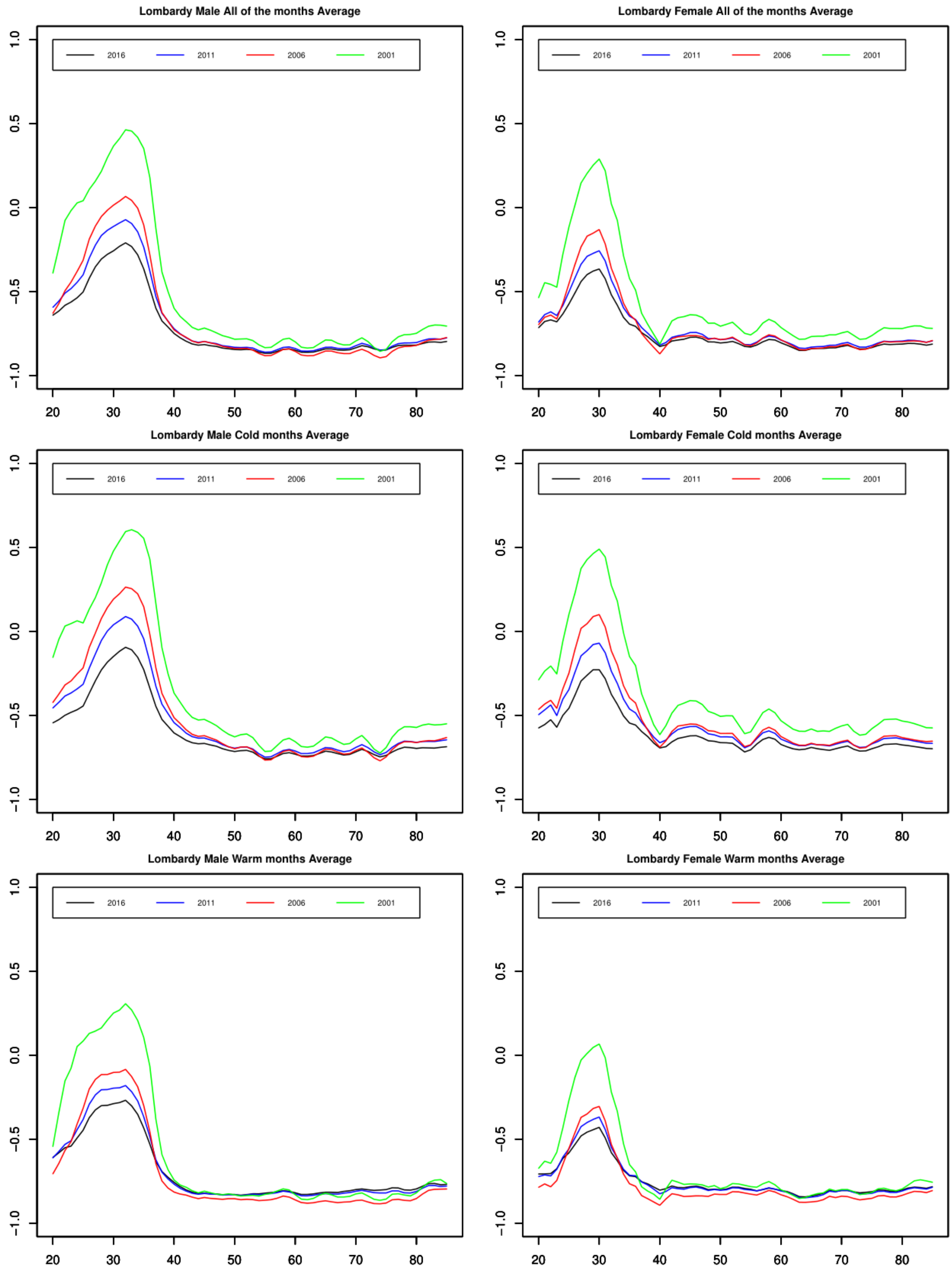

Fig 5.5 – Robustness check of Pearson's correlation coefficient between $\boldsymbol{m}_{x,t}$ and the average temperature for various periods of time for Lombardy, by gender, from 1974 to the reported year.

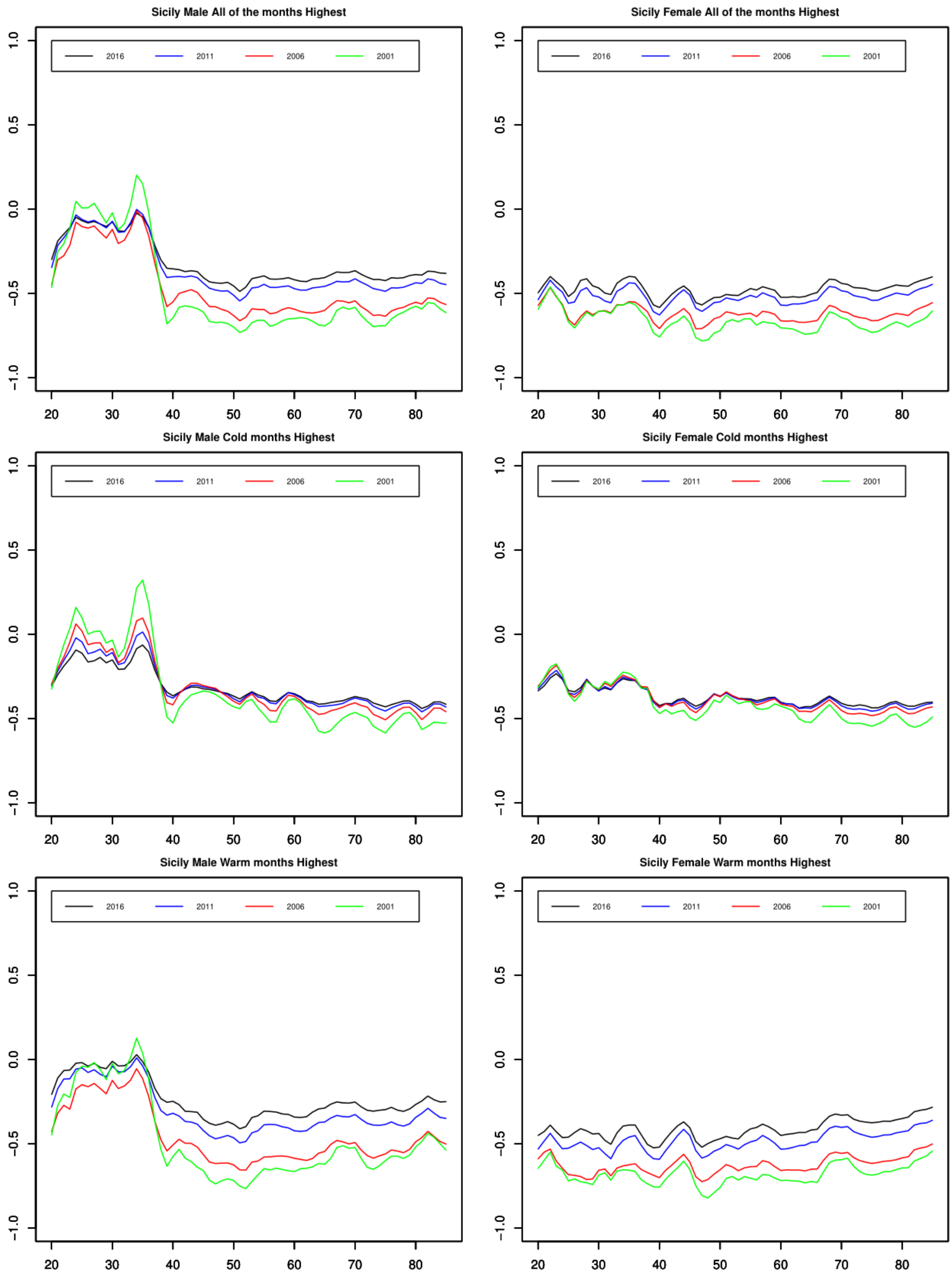

Fig 5.6 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and the highest temperature for various periods of time for Sicily, by gender, from 1974 to the reported year.

Chapter 6

The model

In this chapter, the temperature-related factor model is presented, discussed, analysed and compared with the LC, P and OL models. First, the model is proposed, its factors are singularly explained (section 6.1) and the estimation of its parameters is discussed (section 6.1.1) before fitting them (section 6.1.2). Subsequently, the parametric and projection risks are surveyed. The parametric risk is examined by checking the parameter uncertainty using the bootstrapping technique (section 6.1.3). Projection risk is evaluated by exploring the goodness of fit (section 6.2): firstly by investigating the analysis of the residuals (section 6.2.1), secondly by comparing two error measures to those of the LC, P and OL models (section 6.2.2) and by checking their robustness (section 6.2.1), and thirdly by scanning the same measures for the out-of-sample data (sections 6.3.1 and 6.3.2 for their robustness). After these steps, the model has been forecasted (section 6.3).

6.1 The model

The model proposed by Seklecka, Pantelous and O'Hare[29] is:

$$\ln(m_{x,t})=b_x^1+k_t^1+(\bar{x}-x)\,k_t^2+(\bar{x}-x)^+k_t^3+[(a-x)^+ + ct_x(x-a)^+]^2k_t^4+\gamma_{t-x}+\varepsilon_{x,t} \quad (26)$$

Where:

$\boldsymbol{b_x^1}$ reflects the general mortality path by age, as well as the LC model's general mortality path.

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[29] Seklecka M, Pantelous AA, O'hare C. Mortality effects of temperature changes in the United Kingdom. *Journal of Forecasting*. 2017; **36**: 824-841. https://doi.org/10.1002/for.2473.

$\boldsymbol{k_t^1}$, $\boldsymbol{k_t^2}$ and $\boldsymbol{k_t^3}$ describe mortality changes for different calendar years for, respectively, ages, different age classes and younger ages.

$\boldsymbol{\gamma_{t-x}}$ represents the cohort effect.

$(\bar{x} - x)^+ = \max(\bar{x} - x, 0)$ with $\bar{x}$ being the arithmetical mean of the ages considered.

This model preserves the good aspect of the LC model (described in section 3.2) and possesses new positive features as a consequence of the inclusion of other parameters. Furthermore, it may be considered as an extension of the Plat model (described in section 3.3) or the O'Hare and Li model (described in section 3.4). The innovations are the additions of the factor $\boldsymbol{k_t^4}$ and the factor $\mathbf{ct_x}$ for ages after **a**. The purpose of the time-related factor $\boldsymbol{k_t^4}$ is to capture some of the temperature effects and non-linear features. The factor $\mathbf{ct_x}$ refers to the relationship between the temperature and the central mortality rates at age **x** and it is calculated as the Pearson's correlation coefficient between these two variables. By observing the formula (26), it can be seen that $\mathbf{ct_x}$ includes temperature fluctuations only for elderly ages (above **a**) in a non-linear approach. Seniors are known to be the group with the most exposure to temperature changes (see Chapter 1). The selection of age **a** is discretionary; it may be selected by analysing the Pearson's correlation coefficient between temperature and central mortality rates, which generally becomes stable and strong from around age 40-45 years (see sections 5.3 and 5.3.1) for the Lazio, Lombardy and Sicily datasets over the period. Moreover, a single value for **a** or a different value for each region, each temperature series, each period of the year or even each dataset may be considered. On the one hand, a very general value for **a** might worsen the fitting quality; on the other hand, particularising too much might cause the general validity of the model to be lost. In this thesis, however, it is decided to calculate a single age threshold[30] in order to give more

[30] The work of Seklecka, Pantelous and O'Hare considers only the UK population and the average temperature, so their possibilities were limited. They calculated a general **a** without distinguishing between all, cold or warm months.

general worth to the model and because, for these data, the worsening is not significant. Age **a** is indicatively chosen to make the model sensitive to $\boldsymbol{ct_x}$ when the survival curve starts to decrease rapidly. Therefore, after having compared the results of different **a** values for the range $[55, 65]$, the age of 58 is selected for all the datasets since it returns the best results in terms of MAPE[31] (described further in section 6.2.2).

6.1.1 Estimation of parameters

This model includes six stochastic factors and it has identifiability problems just as the other stochastic mortality models presented in Chapter 3; hence, different parameterisations may produce the same central mortality rate. The model has the same time series structure of the P model (*2009*) and the OL model (*2012*); thus, the following parameter transformations do not change the results:

$$\begin{cases} \widetilde{\boldsymbol{b}}_x^1 = \boldsymbol{b}_x^1 + \boldsymbol{\psi}_1 - \boldsymbol{x}\boldsymbol{\psi}_2 + \boldsymbol{x}^2\boldsymbol{\psi}_3 \\ \widetilde{\boldsymbol{k}}_t^1 = \boldsymbol{k}_t^1 + \boldsymbol{t}\boldsymbol{\psi}_2 + (\boldsymbol{t}^2 - 2\overline{\boldsymbol{x}}\boldsymbol{t})\,\boldsymbol{\psi}_3 \\ \widetilde{\boldsymbol{k}}_t^2 = \boldsymbol{k}_t^2 + 2\boldsymbol{t}\boldsymbol{\psi}_3 \\ \widetilde{\boldsymbol{\gamma}}_{t-x} = \boldsymbol{\gamma}_{t-x} - \boldsymbol{\psi}_1 - (\boldsymbol{t}-\boldsymbol{x})\boldsymbol{\psi}_2 - (\boldsymbol{t}-\boldsymbol{x})^2\boldsymbol{\psi}_3 \end{cases} \tag{27}$$

and

$$\begin{cases} \widetilde{\boldsymbol{b}}_x^1 = \boldsymbol{b}_x^1 + \boldsymbol{c}_1 + \boldsymbol{c}_2(\overline{\boldsymbol{x}} - \boldsymbol{x}) + \boldsymbol{c}_3(\overline{\boldsymbol{x}} - \boldsymbol{x})^+ \\ \widetilde{\boldsymbol{k}}_t^1 = \boldsymbol{k}_t^1 - \boldsymbol{c}_1 \\ \widetilde{\boldsymbol{k}}_t^2 = \boldsymbol{k}_t^2 - \boldsymbol{c}_2 \\ \widetilde{\boldsymbol{k}}_t^3 = \boldsymbol{k}_t^3 - \boldsymbol{c}_3 \end{cases} \tag{28}$$

With $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \boldsymbol{\psi}_3$, $\boldsymbol{c}_1$, $\boldsymbol{c}_2$ and $\boldsymbol{c}_3$ constants, which may be resolved by setting identifiability constraints:

[31] In other words, it is the value by which the model is more times preferable to the LC, P and OL models.

$$\begin{cases} \sum_t k_t^1 = 0 \\ \sum_t k_t^2 = 0 \\ \sum_t k_t^3 = 0 \\ \sum_t k_t^4 = 0 \\ \sum_{c=c_0}^{c_1} \gamma_c = 0 \\ \sum_{c=c_0}^{c_1} c\gamma_c = 0 \\ \sum_{c=c_0}^{c_1} c^2\gamma_c = 0 \end{cases} \tag{29}$$

Where:

$c = t - x$.

c_0 and c_1 are the earliest and the latest year of birth used to fit the cohort effect. The first four constraints help to normalise the estimates of the period indexes and may be imposed by applying the transformation (28) with:

$$c_i = \frac{1}{n}\sum_t k_t^i \qquad\qquad i=1,2,3 \tag{30}$$

The last three constraints enable the γ_{t-x} process to consider only the cohort effect which fluctuates around zero without linear or quadratic trend and without compensating for age or calendar year effects; they may be imposed by applying the transformation (27) with the constants obtained by the regression[32] of γ_{t-x} on $(t-x)$ and $(t-x)^2$:

$$\gamma_{t-x} = \psi_1 + (t-x)\psi_2 + (t-x)^2\psi_3 + \varepsilon_{x,t} \qquad \varepsilon_{x,t} \sim N(0,\sigma^2) \text{ i.i.d.} \tag{31}$$

6.1.2 Model fitting

With the aim of taking into account the observable heteroscedasticity of the mortality data, especially for the elderly ages, the Brouhns, Denuit and Vermunt (*2002*) fitting methodology variation (already mentioned in section 3.2.3) is used, instead of the original Lee and Carter proposal of employing the

[32] For more information, see *Haberman and Renshaw (2011)*, Appendix A.

Singular Value Decomposition (SVD). This variant confers more realism on the model and more accurately forecasts mortality, and it has been used by many other authors, including Renshaw and Haberman *(2006)*, Cairns et al. *(2009)*, Plat *(2009)*, and O'Hare and Li *(2012)*[33].

Therefore, the number of deaths $D_{x,t}$ is modelled as a Poisson distribution with parameter $\boldsymbol{E_{x,t}\, m_{x,t}}$, where $\boldsymbol{E_{x,t}}$ is the number of exposed to risk and $\boldsymbol{m_{x,t}}$ is the central mortality rates to be estimated[34]:

$$D_{x,t} \sim Poisson\ (E_{x,t}\, \mu_{x,t}) \tag{14}$$

Finally, the parameters are estimated by maximising the log-likelihood function[35]:

$$L\,(\psi, D, E) = \sum_{x,t} D_{x,t}\, ln\left[E_{x,t} m_{x,t}(\Phi)\right] - E_{x,t} m_{x,t}(\psi) - ln(D_{x,t}!) \tag{17}$$

Figures 6.1, 6.2, 6.3 and 6.4 plot the estimated values of $\boldsymbol{k_t^1}$ (a), $\boldsymbol{k_t^2}$ (b), $\boldsymbol{k_t^3}$ (c) and $\boldsymbol{k_t^4}$ (d), for different datasets, by changing the variables (period of the year, temperature series, gender and region, respectively). Very generally speaking, an underlying shape is observable for the parameters. The parameter $\boldsymbol{k_t^1}$ shows a decreasing and almost linear trend for all datasets, in line with the expectation. Parameter $\boldsymbol{k_t^2}$ seems to be the parameter most sensitive to data changes, and it is more difficult to describe it in an overall view. Parameter $\boldsymbol{k_t^3}$ rises for the first 20-25 years and then decreases, while $\boldsymbol{k_t^4}$ behaves in the opposite fashion; however, their shape is not similar to a parabola, but rather

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33 For more details, please see Chapter 3.

34 In the original paper, Brouhns et al. used the force of mortality $\mu_{x,t}$ instead of the central mortality rates $\boldsymbol{m_{x,t}}$. Nonetheless, they have been noted to coincide by supposing a constant force of mortality over the period.

35 Instead of using the classic Newton-Raphson iterative procedure, a standard statistical software package is used (see package "gnm", Turner and Firth, (*2015*)) since, according to Currie (2016), many stochastic mortality models may be considered as generalized linear or non-linear models. This is also the approach of the StMoMo package (Villegas, Millossovich and Kaishev, *(2017)*), which is extensively used in this thesis.

appears erratic and irregular. Eventually, the estimated values of parameters remain very similar when either the data's period of the year (Fig. 6.1) or the temperature series (Fig 6.2) is modified, while they are somewhat sensitive to the gender variable (Fig 6.3), and to the region from which the data are sampled (Fig 6.4), notwithstanding the general shape which remains.

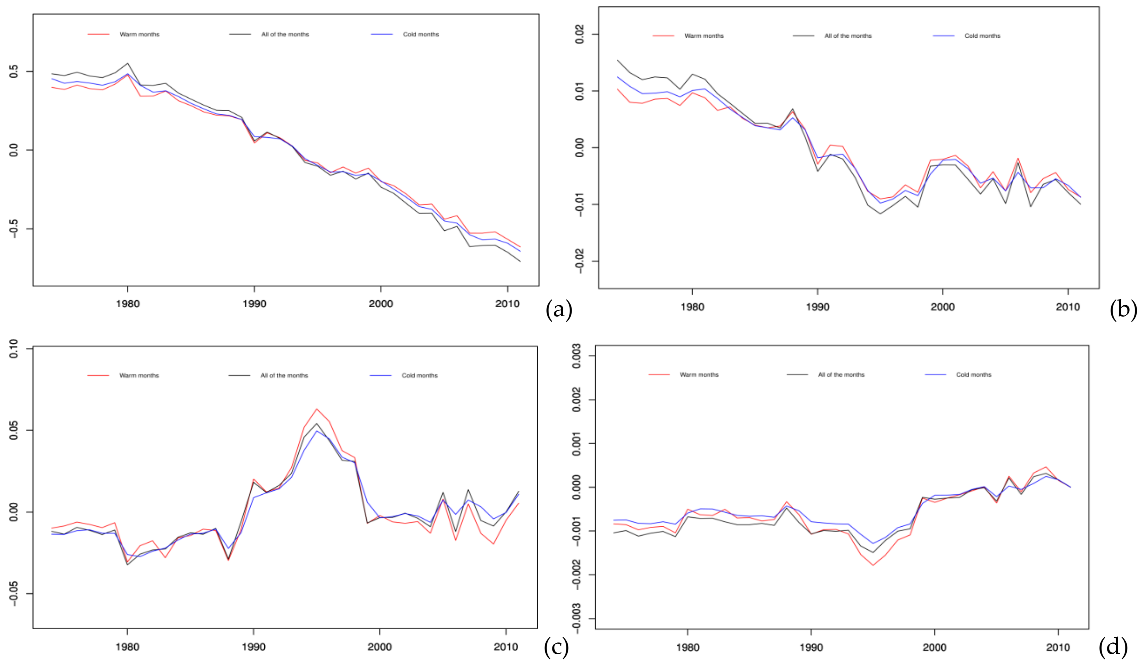

Fig. 6.1 - Estimated values of factors, data for males from Lazio considering the highest temperature.
(a) - $\boldsymbol{k}_t^1$, (b) - $\boldsymbol{k}_t^2$, (c) - $\boldsymbol{k}_t^3$, (d) $\boldsymbol{k}_t^4$.

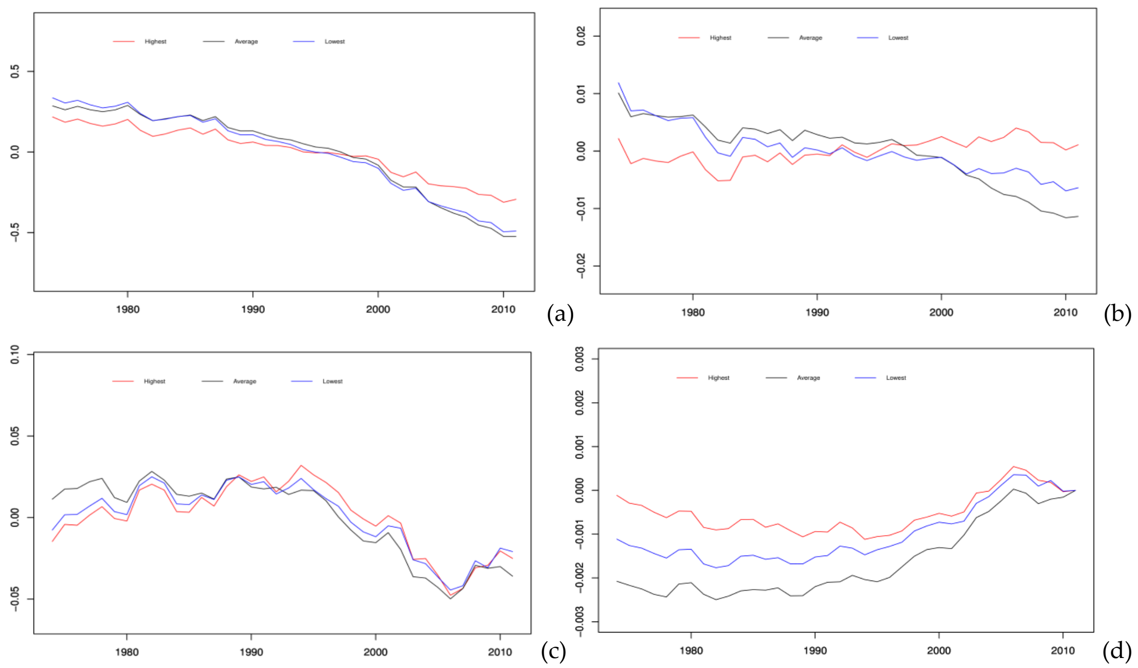

Fig. 6.2 – Estimated values of factors, cold months data for males from Sicily.
(a) - $\boldsymbol{k}_t^1$, (b) - $\boldsymbol{k}_t^2$, (c) - $\boldsymbol{k}_t^3$, (d) $\boldsymbol{k}_t^4$.

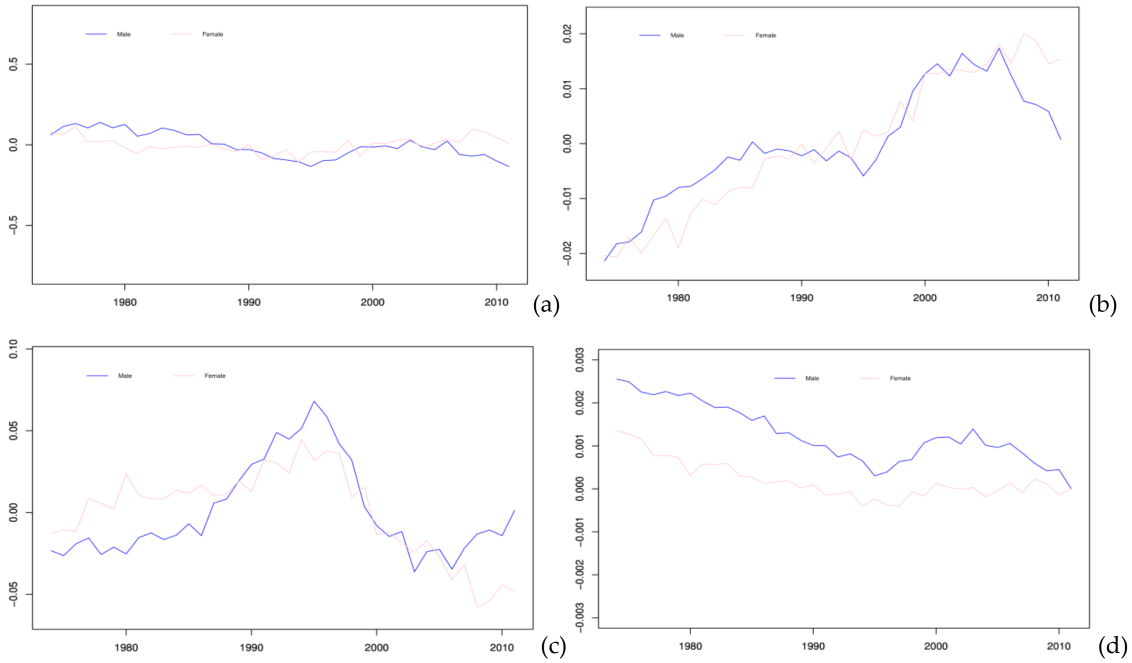

Fig. 6.3 – Estimated values of factors, warm months data from Lombardy, considering the average temperature. (a) - $\boldsymbol{k_t^1}$, (b) - $\boldsymbol{k_t^2}$, (c) - $\boldsymbol{k_t^3}$, (d) $\boldsymbol{k_t^4}$.

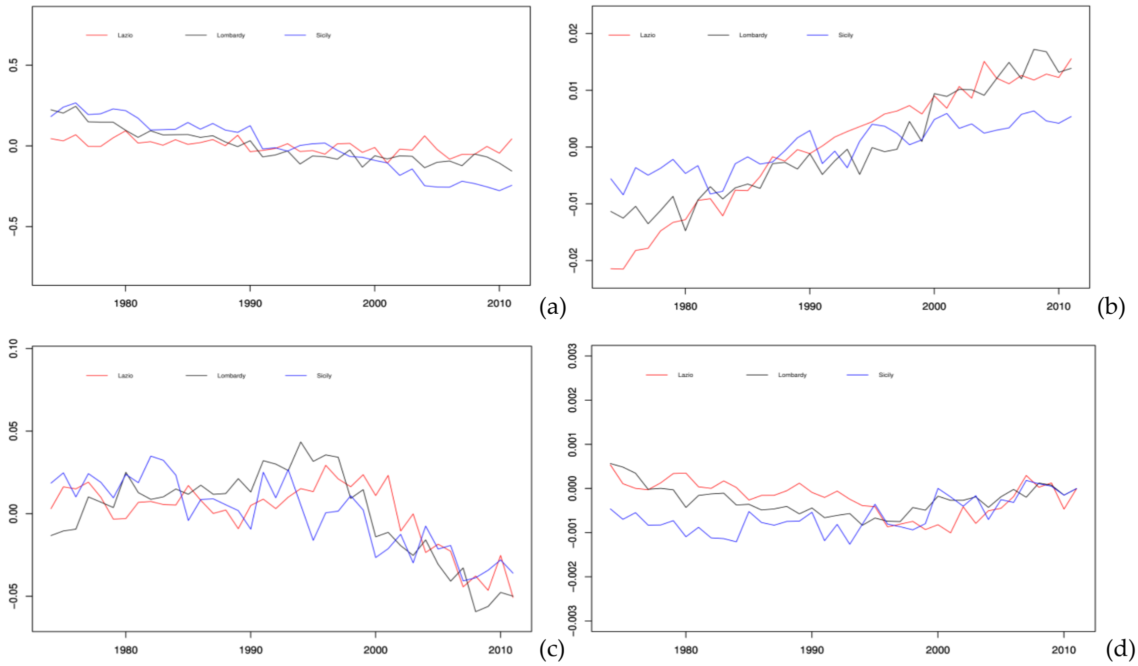

Fig. 6.4 – Estimated values of factors, annual data for females considering the lowest temperature. (a) - $\boldsymbol{k_t^1}$, (b) - $\boldsymbol{k_t^2}$, (c) - $\boldsymbol{k_t^3}$, (d) $\boldsymbol{k_t^4}$.

6.1.3 Parameter uncertainty

Studying the parameters' uncertainty, in order to control the parametric risk, is very important when using Italian regional data because the population numbers are not very large. Therefore, due to the analytical intractability of the stochastic mortality models, the "semi-parametric bootstrap" method proposed by *Brouhns et al., (2005)* is employed: the bootstrapped parameters $\boldsymbol{b_x^{1(b)}}$, $\boldsymbol{k_t^{1(b)}}$, $\boldsymbol{k_t^{2(b)}}$, $\boldsymbol{k_t^{3(b)}}$, $\boldsymbol{k_t^{4(b)}}$ and $\boldsymbol{\gamma_{t-x}^{(b)}}$ ($\boldsymbol{b=1, \dots, 500}$) are yielded by re-estimating the model with each bootstrapped sample of the number of deaths $\boldsymbol{d_{x,t}^{(b)}}$ generated by sampling from the Poisson distribution with mean $\boldsymbol{d_{x,t}}$.

In order to show several examples of the bootstrapping results, the parameter of the annual highest temperature for males from Lazio, the average temperature during the cold months for males from Lombardy, and the lowest temperature during the warm months for females from Sicily are plotted in Figures 6.5, 6.6 and 6.7, respectively[36][37]. The uncertainty of the parameters $\boldsymbol{b_x^1}$, $\boldsymbol{k_t^1}$ and $\boldsymbol{\gamma_{t-x}}$, which are the most "common" among the stochastic mortality models, is slight. With respect to the $\boldsymbol{k_t^2}$, $\boldsymbol{k_t^3}$ and mostly $\boldsymbol{k_t^4}$ parameters, the fan of the simulations is wider and indicates that the parametric risk may add more uncertainty to the projection of the central mortality rates $\boldsymbol{m_{x,t}}$ (see section 6.3.1 and Fig. 6.17). In any event, the results appear satisfactory.

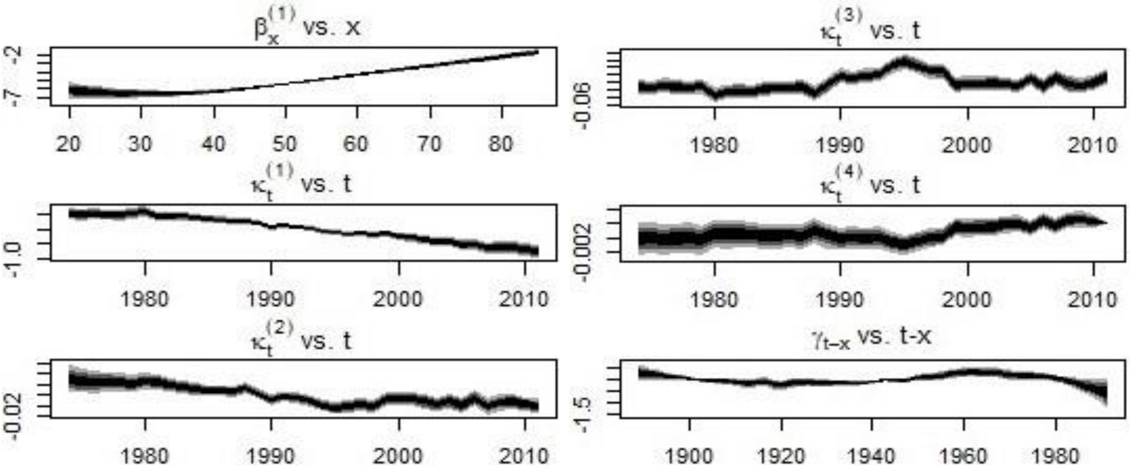

Fig. 6.5 – Bootstrapped parameters of the model for annual highest temperature, for males from Lazio.

[36] Shades in the fan represent the confidence intervals at the 50%, 80% and 95% levels.

[37] Plots of all the other cases are available upon request.

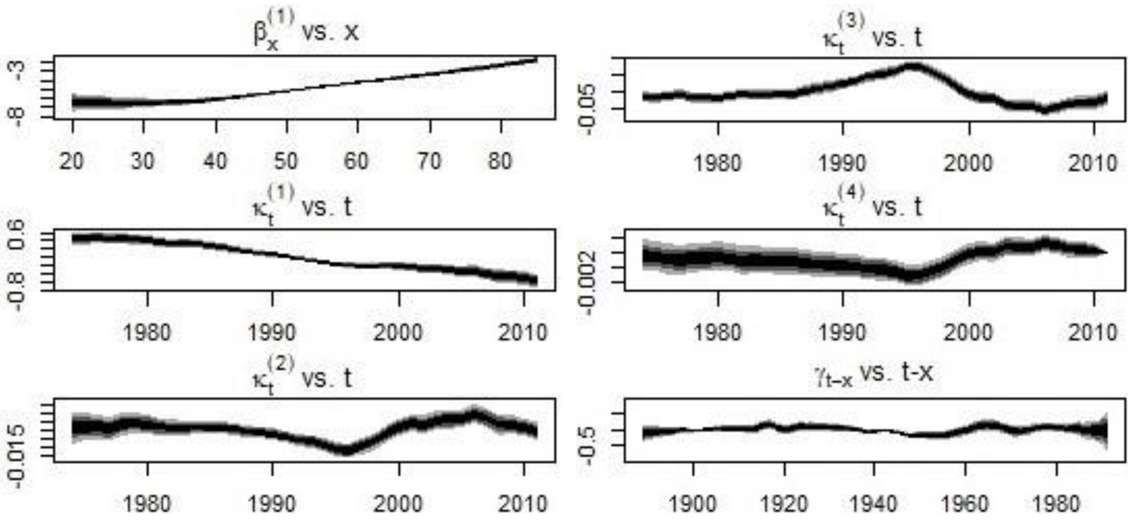

Fig. 6.6 – Bootstrapped parameters of the model for the average temperature during cold months, for males from Lombardy.

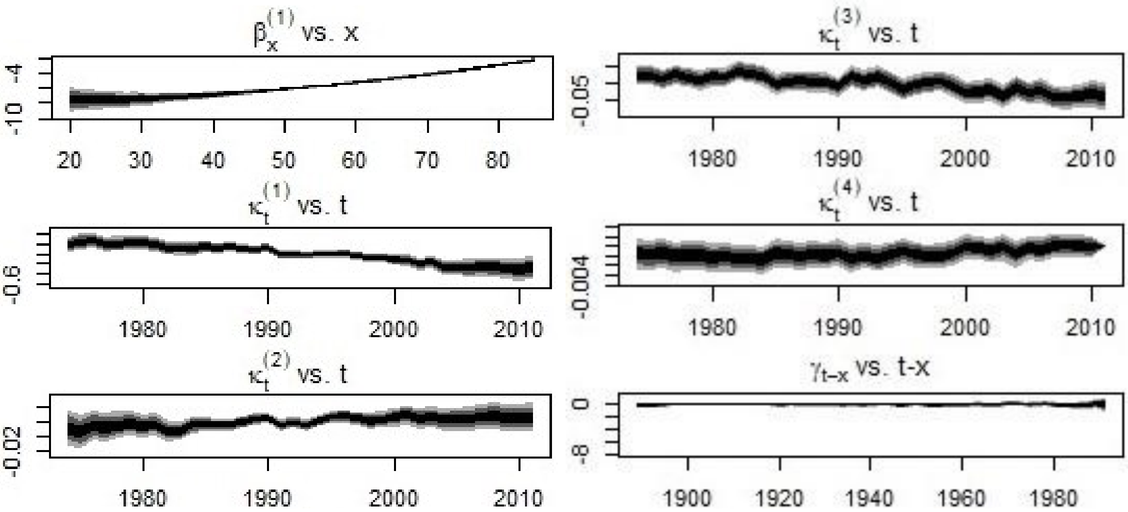

Fig. 6.7 – Bootstrapped parameters of the model for the lowest temperature during warm months, for females from Sicily,

6.2 Goodness of fit

The goodness of fit is investigated first by scanning the analysis of the residuals (section 6.2.1), and secondly by calculating two error measures (also comparing the results with those of the LC, P and OL models (section 6.2.2) and by checking their robustness (section 6.2.2.1).

6.2.1 Analysis of the residuals

Inspecting the residuals of the fitted model is essential to check its fitting quality. Thus, the scaled deviance residuals are analysed:

$$\boldsymbol{r_{x,t} = sign(d_{x,t} - \widehat{d}_{x,t})\sqrt{\frac{dev(x,t)}{\widehat{\phi}}}} \tag{32}$$

Where:

$$\boldsymbol{dev(x,t) = 2\left[d_{x,t}\, ln\left(\frac{d_{x,t}}{\widehat{d}_{x,t}}\right) - (d_{x,t} - \widehat{d}_{x,t})\right]} \tag{33}$$

$$\boldsymbol{\widehat{\phi} = \frac{D(d_{x,t}, \widehat{d}_{x,t})}{K - v}} \tag{34}$$

$$\boldsymbol{D\left(d_{x,t}, \widehat{d}_{x,t}\right) = \sum_{x,t} dev(x,t)} \tag{35}$$

Where $\boldsymbol{D}$ is the total deviance, $\boldsymbol{v}$ is the number of parameters and $\boldsymbol{K}$ is the number of observations.

In order to exhibit several examples, scatter plots of the residuals for males and females of the same samples shown in the previous section are plotted below (Figures 6.8, 6.9, 6.10, 6.11, 6.12 and 6.13)[38]. Except for the dataset displayed in Figure 6.11, where the graphs are not adequate, the analysis of the residuals shows a suitable goodness of fit considering the results which may be found in the literature for the other stochastic mortality models[39]. The scatter plots of the residuals by age often give good results, even if their variability sometimes rises for older ages (Figures 6.8 (a), 6.12 (a), and 6.13 (a)). The residuals also seem random along the calendar years. Even the scatter plots by year of birth display fairly random residuals, although a larger variability may emerge for the first-half of the cohorts (Figures 6.8 (c), 6.12 (c) and 6.13 (c)).

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[38] All other plots of the analysis of the residuals are available upon request.

[39] The scatter plots of the residuals have been largely used in *Haberman and Renshaw (2011)*, for instance.

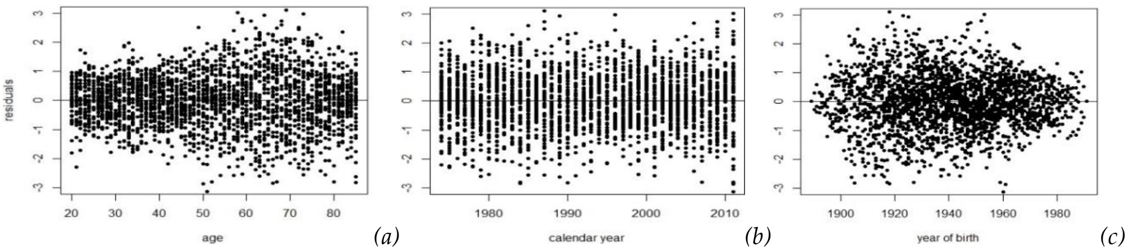

Fig. 6.8 – Scatter plots of the residuals for females from Lazio, annual highest temperature.

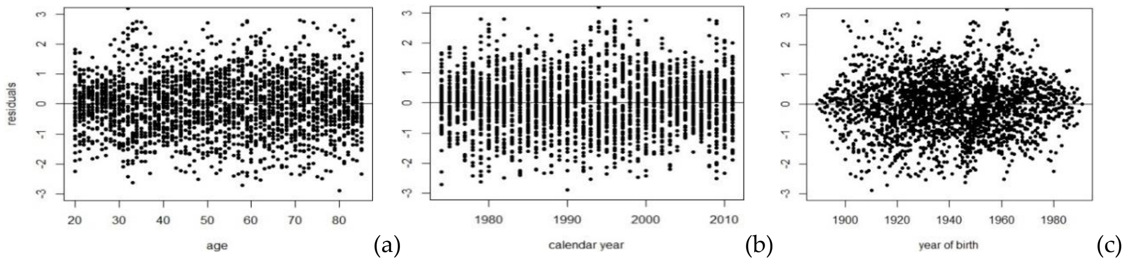

Fig. 6.9 – Scatter plots of the residuals for males from Lazio, annual highest temperature.

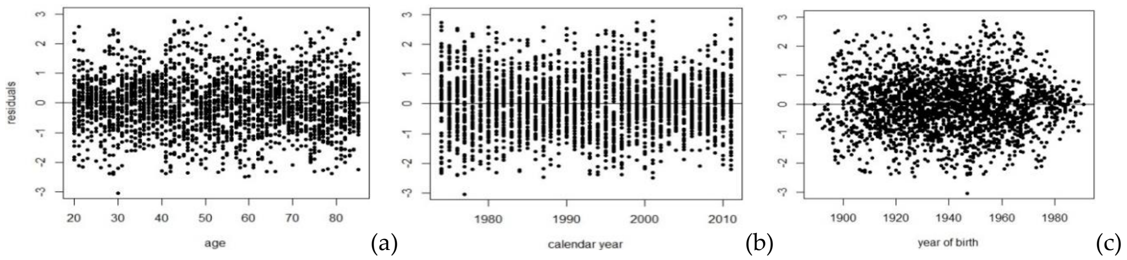

Fig. 6.10 – Scatter plots of the residuals for females from Lombardy, average temperature during cold months.

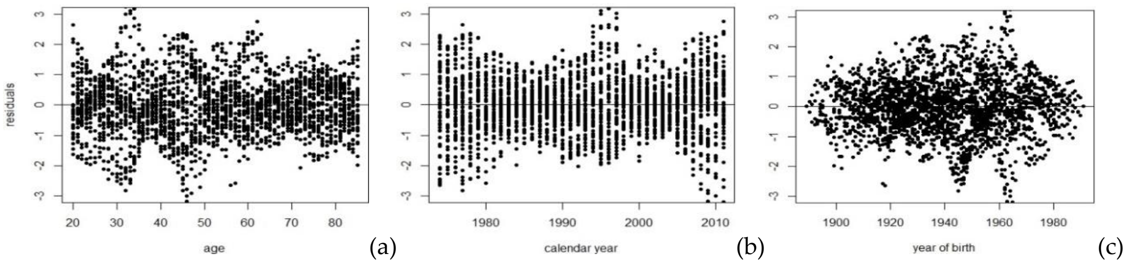

Fig. 6.11 – Scatter plots of the residuals for males from Lombardy, average temperature during cold months.

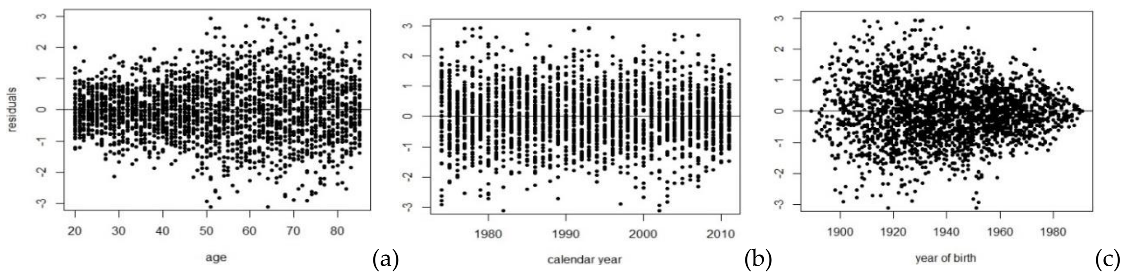

Fig. 6.12 – Scatter plots of the residuals for females from Sicily, lowest temperature during warm months.

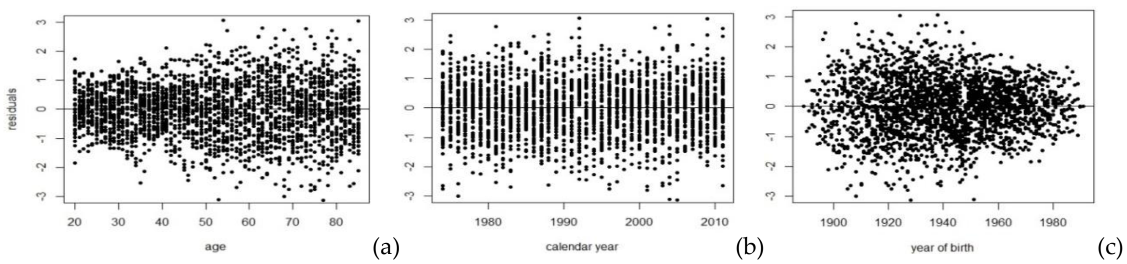

Fig. 6.13 – Scatter plots of the residuals for males from Sicily, lowest temperature during warm months.

6.2.2 Fitting quality measures

A good model might give estimated mortality rates which do not differ much from those observed in the fitted period (1974-2011). Therefore, two error measures are calculated to scan the fitting quality of the proposed model, and the results are compared with those from the LC, P and OL models. The Mean Absolute Percentage Error (MAPE) refers to the absolute fitting quality, while Akaike's Information Criterion (AIC) considers the number of parameters, penalising the most complicated models. MAPE is defined as:

$$MAPE = \frac{100}{NM} \sum_{x,t} \left| \frac{m_{x,t} - \widehat{m_{x,t}}}{m_{x,t}} \right| \tag{36}$$

Where **N** (**N**=38) is time dimension and **M** (**M**=66) is age dimension.
AIC is performed[40] to discover whether a better model fitting in terms of MAPE derives from over-parameterisation. AIC is defined as:

$$AIC = 2k - 2L(\psi) \tag{37}$$

Where k is the number of parameters being estimated and L (ψ) is the log-likelihood of the estimated parameter ψ[41].

Findings are shown in Table 6.1 for MAPE and in Table 6.2 for AIC. For each dataset, the name of the expected best proposed model (among the ones related to the highest, average and lowest temperatures), in line with the findings of the correlation between $m_{x,t}$ and temperature changes, is written in italics in order to discover whether it also gives the best fitting results[42] (see section 5.3). Moreover, each proposed model which is preferable to the LC, P

[40] The MAPE functions from the R package "MLmetrics" (for more details, see: https://cran.r-project.org/web/packages/MLmetrics/), and the AIC function from the R package "StMoMo" (for more details, see: https://cran.r-project.org/web/packages/StMoMo/) are applied.

[41] In particular, k is 178, 277, 277 and 314 for LC, P, OL and the proposed model, respectively.

[42] This logical expectation is, once again, supported by the findings of Secklecka, Pantelous and O'Hare.

and OL models is indicated by boldface, and the absolute best one is highlighted.

With respect to MAPE, better fitting is observed for all the proposed models as compared to the LC, P and OL models[43]. Apart from the LC model, which gives much higher errors, the average difference from the proposed model MAPE compared to the P and OL models is 0.06% for males and 0.05% for females. Regionally speaking, the average improvement is 0.04% for Lazio, 0.07% for Lombardy and 0.05% for Sicily. Hence, the outputs indicate that the proposed model is a better fit for all these data. The next step is to analyse the AIC, since the model has many more parameters than the others, in particular the LC model.

By using AIC, the proposed model loses its highest ranking for some datasets. Once again, dividing Italy into three regions has been revealed to be a good decision: Lombardy appears to be the region which benefits most from the proposed model, it being always the best choice for this region; moreover, for Lombardy, every proposed model is preferable to the LC, P and OL models for the warm months and all months. With respect to Sicily, the worst AIC results are given for the proposed model, showing the different behaviour[44]. The region of Lazio, finally, shows intermediate results that a proposed model is always selected for males, but never for females. In any case, the proposed model always gives the best AIC outputs compared to those of the LC model, and the differences between both the P and the OL models are always negligible; thus, the proposed model continues to represent a fine alternative.

To summarise, in terms of MAPE, a better fitting quality is observed for all of the temperature-related factor models compared with the LC, P and OL models. Notwithstanding, sometimes the improvement is not very significant, considering that these models contain more parameters and are less manageable. In fact, one of the proposed models gives the smallest AIC only

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[43] Except for the lowest temperature during the cold months for males from Lazio.

[44] Except for the results obtained for all of the months for males.

half the time, despite remaining always preferable to the LC model and despite the very small differences with the P and OL models. Above all, and apart from what was expected, the temperature series which are more correlated with the mortality rates $\boldsymbol{m_{x,t}}$ (see section 5.3) are not the ones which give the smallest errors. Moreover, contrary to previous findings, using different temperature series (highest, lowest or average) gives very similar results.

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	1.30%	1.24%	1.28%
		P	0.73%	0.66%	0.72%
		OL	0.71%	0.63%	0.69%
		Proposed - High	**0.67%**	**0.59%**	**0.65%**
		Proposed -Avg	**0.65%**	**0.58%**	**0.64%**
		Proposed - Low	**0.66%**	0.66%	**0.65%**
	Female	LC	1.04%	0.94%	1.04%
		P	0.72%	0.62%	0.72%
		OL	0.74%	0.64%	0.74%
		Proposed - High	**0.70%**	**0.60%**	**0.70%**
		Proposed -Avg	**0.70%**	**0.60%**	**0.70%**
		Proposed - Low	**0.69%**	**0.59%**	**0.69%**
Lombardy	Male	LC	1.66%	1.61%	1.64%
		P	0.85%	0.81%	0.84%
		OL	0.80%	0.76%	0.79%
		Proposed - High	**0.70%**	**0.71%**	**0.75%**
		Proposed - Avg	**0.76%**	**0.72%**	**0.75%**
		Proposed - Low	**0.77%**	**0.72%**	**0.76%**
	Female	LC	1.16%	1.09%	1.14%
		P	0.65%	0.58%	0.64%
		OL	0.69%	0.63%	0.68%
		Proposed - High	**0.60%**	**0.53%**	**0.59%**
		Proposed - Avg	**0.60%**	**0.53%**	**0.59%**
		Proposed - Low	**0.60%**	**0.54%**	**0.59%**
Sicily	Male	LC	1.08%	1.00%	1.06%
		P	0.67%	0.60%	0.66%
		OL	0.68%	0.60%	0.67%
		Proposed - High	**0.64%**	**0.56%**	**0.56%**
		Proposed -Avg	**0.64%**	**0.56%**	**0.56%**
		Proposed - Low	**0.61%**	**0.55%**	**0.61%**
	Female	LC	0.99%	0.88%	0.97%
		P	0.72%	0.62%	0.71%
		OL	0.74%	0.74%	0.73%
		Proposed - High	**0.70%**	**0.61%**	**0.69%**
		Proposed -Avg	**0.70%**	**0.60%**	**0.69%**
		Proposed - Low	**0.70%**	**0.61%**	**0.70%**

Table 6.1 – The MAPE of the models for the period 1974-2011.

AIC	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	20254	20118	19832
		P	18744	18680	18466
		OL	18709	18638	18434
		Proposed - High	**18701**	18651	**18432**
		Proposed - Avg	**18682**	**18629**	**18429**
		Proposed - Low	**18716**	18658	18447
	Female	LC	17715	17553	17385
		P	17242	17185	16975
		OL	17275	17213	17004
		Proposed - High	17253	17209	17003
		Proposed - Avg	17278	17225	17020
		Proposed - Low	17295	17240	17028
Lombardy	Male	LC	26053	25798	25498
		P	21386	21300	21122
		OL	21175	21072	20921
		Proposed - High	**21154**	**21063**	**20915**
		Proposed - Avg	**21154**	21079	**20915**
		Proposed - Low	**21162**	21101	**20921**
	Female	LC	19549	19416	19189
		P	18270	18236	18033
		OL	18341	18301	18099
		Proposed - High	**18244**	**18226**	**18013**
		Proposed - Avg	**18240**	**18230**	**18007**
		Proposed - Low	**18240**	18245	**18004**
Sicily	Male	LC	19039	18895	18652
		P	18341	18290	18041
		OL	18357	18302	18055
		Proposed - High	18357	18311	18074
		Proposed -Avg	18383	18329	18088
		Proposed - Low	**18322**	18306	18047
	Female	LC	17375	17284	17004
		P	17087	17055	16774
		OL	17124	17089	16805
		Proposed - High	17111	17101	16807
		Proposed -Avg	17132	17111	16818
		Proposed - Low	17130	17110	16823

Table 6.2 – The AIC of the models for the period 1974-2011.

6.2.2.1 Robustness check

A good model must not be susceptible to the range of data used during the fitting process. Therefore, it is crucial to check the fitting robustness. So far, the models have been fitted by using an age range from 20 to 85 years and the period of 1974-2011; thus, the period of 2011-2016 may be used to compare the estimated forecasting results with the observed mortality rates. Therefore, the sample period is modified to 1974-2016 (43 years), 1974-2006 (33 years) and 1974-2001 (28 years) in order to study the robustness. For each period, the parameters of the proposed, LC, P and OL models are fitted, and the results are compared by using the MAPE (equation 36) and the AIC (equation 37). Tables 6.3 and 6.4 display the outputs of these measures for the 28-year retrospective period, which is the unsteadiest because of its reduced width of data as well as the period which shows the greatest differences, albeit still small, with the period of 1974-2011. Additionally, the findings for 1974-2016 and 1974-2006 are presented in Appendix B, and the analysis below applies to these periods, too.

The MAPE continues to rank the proposed model as the best one, for each range of years[45]; moreover, every proposed model is always preferable to the LC, P and OL models[46]. The AIC continues to prefer the proposed model for the dataset of Lombardy and males of Lazio and Sicily (only for all of the months). By changing the retrospective period, some models sometimes change their position with respect to the P and OL models, but the ranking is not remarkably altered.

In brief, the proposed model also does not change its behaviour by using a different range of data. Therefore, it features robustness. Hence, the same conclusions for the period of 1974-2011 are replicated for all the other periods: a stronger correlation between mortality rates $\boldsymbol{m_{x,t}}$ and temperature changes does

[45] Except for the model for females from Lombardy during the warm months (period of 1974-2001), for which the OL model is preferable.

[46] The exceptions are for Sicily during cold months: the highest temperature-related model for males for the period 1974-2006 and the average for females for the period 1974-2016.

not affect the fitting quality. Moreover, the MAPE and AIC results are virtually identical irrespective of whether the highest, average or lowest temperature is used. Bearing in mind these great fitting findings, the proposed model is forecasted in the next sections, and all the relevant features are investigated and compared with the LC, P and OL models, to determine whether the proposed model maintains such good results.

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	1.14%	1.06%	1.12%
		P	0.70%	0.61%	0.69%
		OL	0.71%	0.61%	0.61%
		Proposed - High	**0.64%**	**0.54%**	**0.61%**
		Proposed - Avg	**0.63%**	**0.55%**	**0.61%**
		Proposed - Low	**0.69%**	**0.54%**	**0.60%**
	Female	LC	0.91%	0.81%	0.90%
		P	0.67%	0.56%	0.66%
		OL	0.67%	0.56%	0.66%
		Proposed - High	**0.65%**	**0.55%**	**0.65%**
		Proposed - Avg	**0.65%**	**0.54%**	**0.65%**
		Proposed - Low	**0.60%**	**0.56%**	**0.65%**
Lombardy	Male	LC	1.37%	1.32%	1.35%
		P	0.70%	0.64%	0.64%
		OL	0.65%	0.59%	0.64%
		Proposed - High	**0.59%**	**0.52%**	**0.59%**
		Proposed - Avg	**0.59%**	**0.52%**	**0.59%**
		Proposed - Low	**0.59%**	**0.50%**	**0.58%**
	Female	LC	0.98%	0.91%	0.96%
		P	0.61%	0.54%	0.60%
		OL	0.64%	0.53%	0.53%
		Proposed - High	**0.56%**	**0.50%**	0.55%
		Proposed - Avg	**0.56%**	**0.50%**	0.55%
		Proposed - Low	**0.56%**	**0.49%**	0.55%
Sicily	Male	LC	1.08%	0.99%	1.06%
		P	0.62%	0.54%	0.61%
		OL	0.61%	0.53%	0.60%
		Proposed - High	**0.57%**	**0.49%**	**0.49%**
		Proposed -Avg	**0.56%**	**0.48%**	**0.55%**
		Proposed - Low	**0.57%**	**0.48%**	**0.56%**
	Female	LC	1.06%	0.94%	1.02%
		P	0.72%	0.62%	0.71%
		OL	0.73%	0.63%	0.72%
		Proposed - High	**0.70%**	**0.61%**	**0.69%**
		Proposed -Avg	**0.70%**	**0.59%**	**0.69%**
		Proposed - Low	**0.70%**	**0.61%**	**0.69%**

Table 6.3 – The MAPE of the models for the period 1974-2001.

AIC	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	14729	14580	14446
		P	13920	13851	13715
		OL	13937	13937	13730
		Proposed - High	**13896**	**13833**	**13689**
		Proposed - Avg	**13889**	**13844**	**13680**
		Proposed - Low	**13880**	**13828**	**13677**
	Female	LC	13094	12987	12860
		P	12841	12803	12645
		OL	13730	12802	12802
		Proposed - High	12852	12852	12670
		Proposed - Avg	12849	12817	12665
		Proposed - Low	12862	12862	12674
Lombardy	Male	LC	18397	18106	18027
		P	15591	15591	15419
		OL	15541	15445	15372
		Proposed - High	**15413**	**15318**	**15254**
		Proposed - Avg	**15416**	**15322**	**15264**
		Proposed - Low	**15414**	**15306**	**15246**
	Female	LC	15048	14960	14768
		P	14070	14057	13890
		OL	14074	14057	13894
		Proposed - High	**14051**	**14057**	**13871**
		Proposed - Avg	**14053**	14065	**13869**
		Proposed - Low	**14053**	14071	**13865**
Sicily	Male	LC	14185	14087	13901
		P	13630	13592	13412
		OL	13630	13589	13412
		Proposed - High	13631	13604	13421
		Proposed -Avg	13637	13610	13423
		Proposed - Low	**13629**	13629	13421
	Female	LC	13096	13011	12809
		P	12794	12766	12559
		OL	12807	12776	12570
		Proposed - High	13421	12790	12571
		Proposed -Avg	12819	12799	12585
		Proposed - Low	12823	12801	12595

Table 6.4 – The AIC of the models for the period 1974-2001.

6.3 Forecasting

When considering the good fitting results, it is sensible to continue the analysis by forecasting future mortality rates. The only parameters to be forecast are the time-related ones as well as the cohort-effect, since the age-dependent factors are constant over time. The parameters $\boldsymbol{k_t^1}, \boldsymbol{k_t^2}, \boldsymbol{k_t^3}, \boldsymbol{k_t^4}$ and $\boldsymbol{\gamma_{t-x}}$ are modelled as AutoRegressive Integrated Moving Average (ARIMA) (***p,d,q***) time-series processes, where the parameters $\boldsymbol{p}$, $\boldsymbol{d}$ and $\boldsymbol{q}$ are non-negative integers, $\boldsymbol{p}$ is the order of the autoregressive model, $\boldsymbol{d}$ is the degree of differencing, and $\boldsymbol{q}$ is the order of the moving-average model:

$$\boldsymbol{\Delta^d \mathrm{Y}_t = \delta_0 + \phi_1 \Delta^d \mathrm{Y}_{t-1} + \cdots + \phi_p \Delta^d \mathrm{Y}_{t-p} + \xi_t + \delta_1 \xi_{t-1} + \cdots + \delta_q \xi_{t-q}} \qquad (38)$$

Where $\boldsymbol{\Delta}$ is the difference operator, $\boldsymbol{\delta_0}$ is the drift parameter, $\boldsymbol{\phi_1, \dots, \phi_p}$ are the autoregressive coefficients with $\boldsymbol{\phi_p \neq 0}$, $\boldsymbol{\delta_1, \dots, \delta_q}$ ($\boldsymbol{\delta_q \neq 0}$) are the moving average coefficients and $\boldsymbol{\xi_t}$ is a Gaussian white noise process with variance σ^2_ε. Following previous studies[47], the cohort index $\boldsymbol{\gamma_{t-x}}$, which is the most difficult to handle (see *Currie (2016)*), is supposed to be independent from the period indexes $\boldsymbol{k_t^1}, \boldsymbol{k_t^2}, \boldsymbol{k_t^3}$ and $\boldsymbol{k_t^4}$. Thus, for each dataset[48], the best ARIMA (***p,d,q***) time series are found and forecasted for 20 years (see, for example, Fig 6.14)[49].

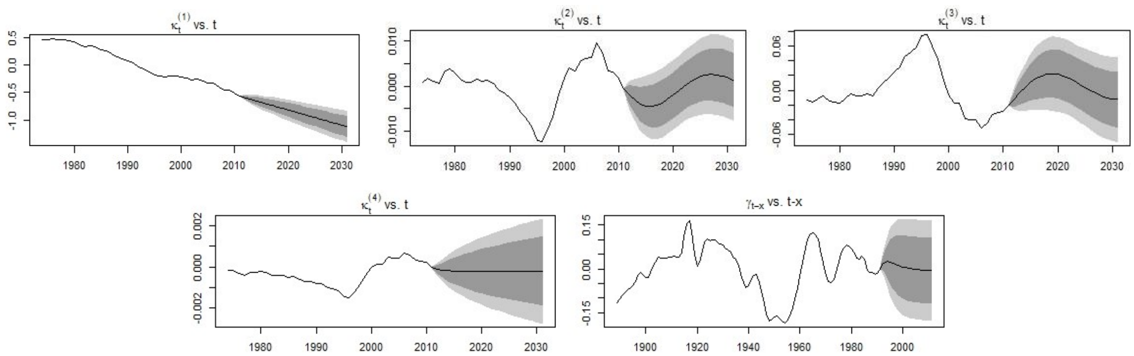

Fig 6.14 – Parameters forecast for Lombardy males, average temperature during cold months.

47 See *Renshaw and Haberman (2006), Cairns et. al (2011),* and *Lovász (2011).*

48 Obviously, different datasets return different ARIMA processes. Thus, the findings of the previous authors for their models over US or UK data might not be the best choice for Italian regional data.

49 The auto.arima function from the R package "forecast" is used, which gives the best ARIMA (***p,d,q***) model according to the AIC (for more details, see: https://cran.r-project.org/web/packages/forecast/).

6.3.1 Simulations analysis

According to *Cairns et al. (2011)*, an easy way to explore the plausibility of the forecasting is to construct fan charts produced by simulations; therefore, 500 simulations for each dataset are computed and analysed. Figure 6.15 shows, as an example, 25 simulations of the parameters and the central mortality rate at age 65 for the annual highest temperature-related model for females from Lazio.

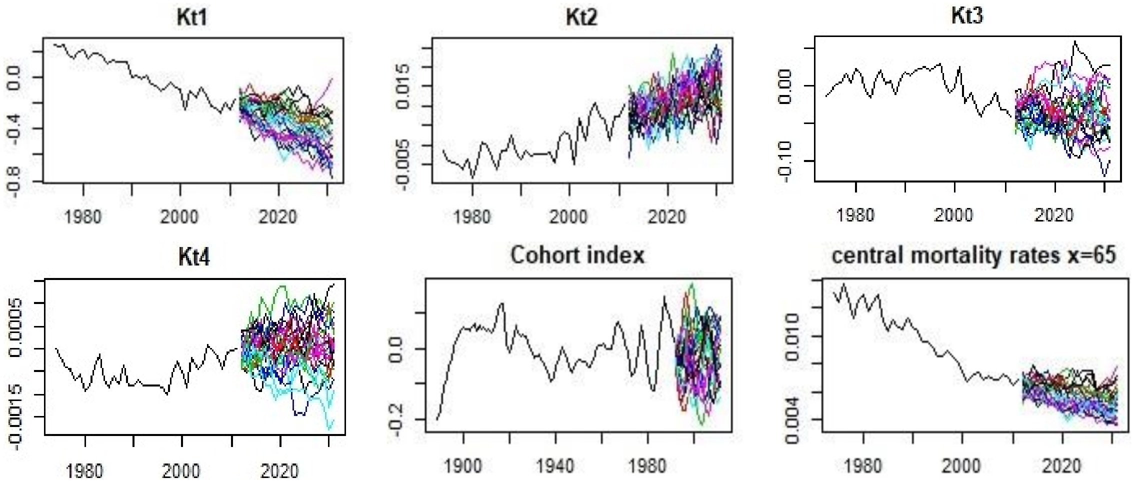

Fig 6.15 – Parameters and central mortality rate simulations for females from Lazio, annual highest temperature.

By observing the fan chart plots[50] of the models for a Sicilian male during the warm months (Fig. 6.16), it is obvious that every model shows different behaviours, and it may be stated that the LC model is not plausible for this dataset, since the fans at age 85 are narrower than at age 65[51], contrary to the historical evidence. The fan charts of the OL model seem to be wider than those of the other models, while, when using the highest or the average temperature, the proposed model gives more downwards prediction than the others, especially for **x**=85.

[50] The fan function from the R package "fanplot" is used (for more details, see: https://cran.r-project.org/web/packages/fanplot/).

[51] This is a well-known issue with the LC model.

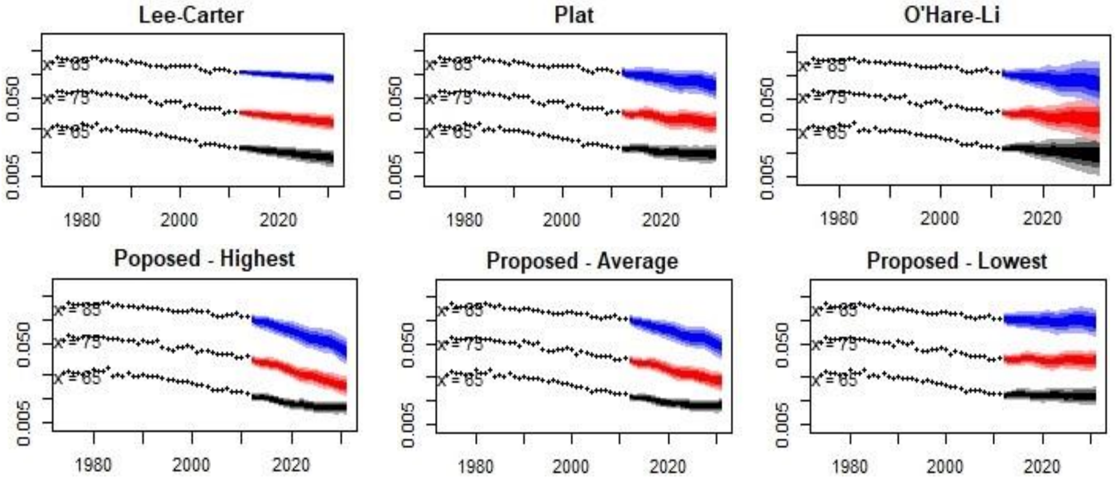

Fig 6.16 – Fan charts of predicted mortality rates at ages 65, 75 and 85 years, for Sicilian male, warm months[52].

Lastly, Figure 6.17 shows that the impact of the parameter uncertainty on the forecast is not properly remarkable. This figure displays the mortality rates of the highest temperature-related proposed model for a 60, 70 and 80-year-old male from Sicily, during the cold months, with a 20-year projection. For the period 1974-2011, dots represent raw data and black lines represent the fitted mortality rates. For the period 2011-2031, dashed lines show central forecast and black dotted lines indicate the 95% prediction intervals without the parameter uncertainty. Lastly, dot-dashed red lines indicate 95% prediction intervals including parameter uncertainty.

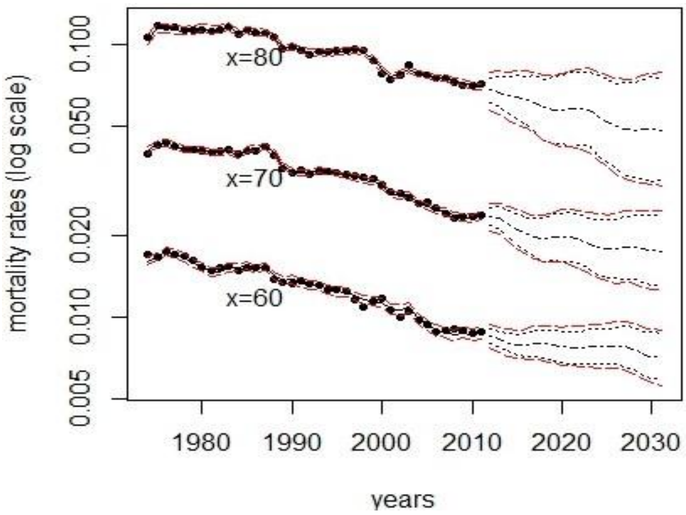

Fig. 6.17 – Prediction of mortality rates for males from Sicily, highest temperature during the cold months.

[52] The dots indicate historical rates from 1974 to 2011. Shades in the fan represent prediction intervals at 50%, 80% and 95% levels.

6.3.2 Backtesting

The main feature of a model's forecasting process is that the projected mortality rates should be as similar as possible to the observed ones. It is possible to evaluate this similarity through backtesting. Table 6.9 shows the MAPE (equation 36) results of the out-of-sample test for each temperature-related factor model, and also compares them with those of the LC, P and OL models. All the models are fitted for the period 1974-2011, and the test concerns the 5-year projection from 2011 to 2016.

The findings for the proposed model show less improvement compared to the in-sample analysis (section 6.2.2). This result is not astonishing since is possible to find many examples in the literature of different rankings of mortality models by using either the in-sample or the out-of-sample test for the same dataset (see, for instance, *Cairns et al. (2011)*).

With respect to the region of Lazio, results for males remain great: the proposed model always gives a smaller MAPE than those of the LC, P and OL models. Moreover, the difference with the LC model is significant[53]. With respect to Lazio females, the proposed model continues to accurately fit the future data for cold months and all of the months by using the highest or the average temperature; nevertheless, the results for the lowest temperature and the warm months are considerably negative, compared to those of the other models.

Unlike Lazio, where the expectations built by observing the in-sample AIC (see Table 6.2) are maintained, Lombardy does not retain them. In fact, while Lombardy is the best region in the AIC in-sample analysis, it emerges as the worst one for the MAPE out-of-sample test. For the highest temperature for males and the lowest one for females, both only during the cold months, the model shows errors in line with those of the P model, which turns out to be the

[53] The LC model gives a 21.55%, 21.71% and 22.01% MAPE for all of the months, cold months and warm months, respectively, while the highest temperature-related factor model, yields 6.28%, 7.43% and 7.03%, respectively.

best model for this region, and the OL model (they remain at approximately 10%). All the other datasets display a MAPE of about 20%; nevertheless, they remain smaller than those of the LC model (except for females during all months)[54].

Lastly, Sicily is the region which displays the most different outcomes, especially for females. The proposed model shows improvements for half of the dataset: for the average temperature during cold months for males and for the lowest temperature during warm months and all of the months for females. However, Sicilian findings are not greater than those of the other regions[55]. Moreover, the LC model is generally the best model for Sicilian females. This suggests that the extensions of the P and OL models, from which the proposed model is constructed and which accurately fit the behaviour of some populations, may not work well for these data.

Ultimately, compared to the LC, P and OL models, the highest temperature shows improvements for 4 of the 18 datasets, the average temperature for 5 of 18 and the lowest temperature for 6 of them; moreover, the proposed models reveal themselves to be the absolute best model 3, 1, and 4 times, respectively. Without considering the LC, P and OL models and only focusing on the difference between the three temperature series, it is revealed that the highest temperature-related model as well as the lowest one show the smallest MAPE for 7 times of 18, while the average temperature-related model exhibits the smallest MAPE for only for 4 of the 18 datasets. Furthermore, the proposed model is superior for 4 over 6 datasets for the cold months, while for 2 for both the warm months and all of the months (which, however, means that the temperature-related factor model is a good competitor of the P and OL models for every period of the year).

Considering the results displayed in Tables 6.10 and 6.11 in the next section, in which the robustness of the results is checked by performing

[54] It is worth noting that the out-of-sample MAPE comparison made by the authors of the proposed model was only performed with the LC model.

[55] Except for the average temperature for females, where the error results are very high.

backtesting for the other two look-ahead periods (from 2006 to 2016 and from 2001 to 2016), it will be possible to draw more general and significant conclusions.

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	21.55%	21.71%	22.01%
		P	7.98%	9.10%	8.96%
		OL	9.08%	10.31%	9.53%
		Proposed - High	**6.28%**	**7.43%**	**7.03%**
		Proposed - Avg	**6.99%**	**7.50%**	**7.58%**
		Proposed - Low	**7.72%**	**6.79%**	**8.44%**
	Female	LC	12.04%	12.71%	12.69%
		P	8.69%	10.54%	9.50%
		OL	7.31%	11.48%	8.27%
		Proposed - High	8.28%	**8.52%**	17.39%
		Proposed - Avg	8.77%	**9.58%**	17.42%
		Proposed - Low	17.15%	16.14%	25.38%
Lombardy	Male	LC	24.59%	25.65%	25.03%
		P	9.34%	10.09%	9.70%
		OL	12.88%	12.70%	13.18%
		Proposed - High	21.06%	10.34%	25.92%
		Proposed - Avg	19.01%	20.98%	25.17%
		Proposed - Low	20.23%	18.72%	20.60%
	Female	LC	20.95%	22.21%	21.39%
		P	9.13%	10.70%	9.12%
		OL	9.22%	13.59%	10.28%
		Proposed - High	22.20%	20.27%	15.52%
		Proposed - Avg	24.13%	21.81%	14.06%
		Proposed - Low	27.90%	**9.80%**	14.65%
Sicily	Male	LC	11.41%	10.96%	12.26%
		P	8.09%	9.12%	8.89%
		OL	5.35%	5.75%	5.82%
		Proposed - High	18.18%	6.29%	15.93%
		Proposed -Avg	10.86%	**5.35%**	11.11%
		Proposed - Low	12.32%	6.53%	8.12%
	Female	LC	8.58%	7.59%	8.82%
		P	8.40%	8.87%	10.23%
		OL	17.03%	12.68%	12.68%
		Proposed - High	26.80%	18.51%	29.67%
		Proposed -Avg	50.90%	60.25%	57.79%
		Proposed - Low	**7.57%**	12.77%	**8.56%**

Table 6.9 – The MAPE of the forecast of the models for the period 2011-2016.

6.3.2.1 Robustness check

In order to check the robustness of the results just presented, and to obtain more data for analysis, Tables 6.10 and 6.11 show the MAPE for the out-of-sample test of the models fitted for 33 years (from 1974 to 2006) and 28 years (from 1974 to 2001), projected for 10 years (from 2006 to 2016) and for 15 years (from 2001 to 2016), respectively.

With respect to the 10 years look-ahead period, the proposed model loses its top ranking for males from Lazio during all months but acquires it for females during the warm months. Thus, the highest temperature-related factor model shows the smallest MAPE for both warm months and cold months for each gender. The situation changes, however, when the horizon projection is increased to 15 years: now the highest temperature model is the best one for all of the months but not for the warm months or the cold months. Considering Lombardy, broadening the horizon period worsens the already bad results. In fact, the proposed model is never chosen, and for the 15 years of projection, the MAPE reaches incredibly high values[56]. With respect to the last region, Sicily, the proposed model with the lowest temperature-related factor always shows fewer errors for females during the period 2006-2016, but never for the period 2001-2016, in which the LC model is selected. For Sicilian males, instead, only the lowest temperature during the warm months provides improvements (for the 15-year period).

To summarise, the results of the proposed model are not very robust, and they often change by modifying the forecast horizon. Moreover, by using the highest, average or lowest temperature, or by analysing the warm months, cold months or all of the months, this may lead to observing illogical vast MAPE differences. Sometimes the proposed model gives good results, but other times

[56] For example, for males, while the OL model gives a MAPE around 20%, the lowest temperature outcomes are 1035.16%, 316.99% and 5339.17% for warm months, cold months and all of the months, respectively.

its error is substantial. That is definitely not a good feature because it makes the model unusable for pricing financial instruments linked to human lives.

Now it is possible to draw more general conclusions with respect to the last considerations of the previous section, which analysed for the period 2011-2016 which temperature series, among the highest, the average and the lowest, was more meaningful for implementation in a stochastic mortality model and for which period of the year the proposed model showed more improvements.

In total, the highest temperature shows improvements compared to the LC, P and OL models, for 11 datasets over 54 (20%), with the average temperature showing improvements for 6 (11%) and the lowest for 10 (19%) of them. Moreover, the highest temperature-related model turns out to be the best model, including of the LC, P and OL models, for 9 (17%) datasets, while the lowest temperature-related model was the best for 7 (13%) of them and the average temperature-related model was best only for 2 (4%) datasets. These results indicate that the highest and lowest temperatures should both be considered in further analyses since it almost never occurs that both show improvements for the same dataset[57]. Furthermore, by only investigating the ranking for these three temperature-related models – that is, by excluding the LC, P and OL models – the highest temperature-related factor dominates the others 20 times over 54 (37%), the lowest one for 23 times (43%) and the average one for only 11 (20%).

With regard to the regions, the differences among them are evident. The proposed model improves the forecast quality of the mortality rates for Lazio 11 times over 18 (61%)[58], for Sicily 7 times (39%)[59], and for Lombardy only 1 time (6%). This result is absolutely inconsistent with the others obtained so far.

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[57] The only exception is represented by males from Lazio, in which case, however, every proposed model is preferable to the others.

[58] In particular, the highest temperature is selected for all the 61% of the times and is almost always preferable than the average or the lowest temperatures, when they are all selected.

[59] With regard to Sicily, the lowest temperature improves the forecast of the $\boldsymbol{m_{x,t}}$ for 6 times over 18 (33%), and the average or the highest temperature are never selected when the lowest is.

The small difference between males, where the proposed model performs better for 8 times over 27 (30%), and female, where it performs better 11 times (41%) entirely derives from the region of Sicily (in which the values are 2 times over 7 (29%) for males and 5 times (71%) for females).

Finally, by inspecting the differences between warm months, cold months, and all of the months, a proposed model is selected for 8 times over 18 (44%) during warm months, for 6 (33%) times during cold months and for 5 (28%) times during all of the months. Besides, there is no relevant relationship between the different periods of the year and the use of the highest, the average or the lowest temperature-related model; except for the average temperature and the cold months (such as the findings of Seklecka, Pantelous and O'Hare have noted).

Therefore, the differences which emerge considering different periods of the year are not very remarkable and the analysis, at least for Italy, might refer to the entire year. A direct consequence of these results is that this model may be well-suited for many countries. The most important finding is that the highest and the lowest temperatures are much more significant for implementation in a mortality model with respect to the average one, in accordance with the studies showing the correlation between mortality and extreme temperatures. That is an absolute novelty in the literature and suggests that this analysis should be intensified in further studies.

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	22.45%	22.21%	22.37%
		P	11.53%	8.66%	12.13%
		OL	12.37%	11.20%	11.39%
		Proposed - High	18.94%	**7.93%**	**9.15%**
		Proposed - Avg	20.35%	26.16%	26.85%
		Proposed - Low	38.40%	83.43%	42.26%
	Female	LC	13.54%	13.39%	13.71%
		P	13.08%	19.65%	15.77%
		OL	13.40%	14.13%	13.55%
		Proposed - High	14.24%	**9.64%**	**11.13%**
		Proposed - Avg	13.20%	17.92%	18.85%
		Proposed - Low	123.28%	139.45%	522.40%
Lombardy	Male	LC	32.43%	31.94%	32.63%
		P	47.30%	44.91%	48.49%
		OL	20.80%	18.47%	20.72%
		Proposed - High	49.19%	37.14%	55.17%
		Proposed - Avg	35.51%	42.89%	50.99%
		Proposed - Low	33.92%	684.51%	39.14%
	Female	LC	25.51%	24.59%	25.41%
		P	13.90%	12.46%	12.68%
		OL	9.62%	9.26%	9.63%
		Proposed - High	21.86%	12.20%	17.33%
		Proposed - Avg	21.22%	11.97%	16.46%
		Proposed - Low	16.50%	12.00%	16.01%
Sicily	Male	LC	14.13%	13.53%	14.04%
		P	7.77%	7.25%	7.85%
		OL	17.51%	13.22%	17.71%
		Proposed - High	91.54%	21.68%	85.89%
		Proposed -Avg	183.42%	10.71%	151.95%
		Proposed - Low	45.67%	17.24%	63.50%
	Female	LC	9.57%	8.94%	9.35%
		P	8.73%	8.41%	8.98%
		OL	16.89%	16.45%	18.15%
		Proposed - High	17.98%	20.43%	19.33%
		Proposed -Avg	57.09%	45.15%	45.33%
		Proposed - Low	**8.19%**	**7.05%**	**8.72%**

Table 6.10 – The MAPE of the forecast of the models for the period 2006-2016.

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	32.38%	32.61%	32.29%
		P	15.09%	12.31%	15.08%
		OL	14.58%	15.16%	16.97%
		Proposed - High	**14.55%**	21.21%	19.06%
		Proposed - Avg	16.46%	17.79%	35.10%
		Proposed - Low	76.73%	23.39%	127.66%
	Female	LC	21.48%	21.20%	21.81%
		P	16.84%	14.96%	18.54%
		OL	31.34%	30.09%	37.23%
		Proposed - High	**11.96%**	**9.90%**	20.94%
		Proposed - Avg	18.77%	**7.22%**	25.19%
		Proposed - Low	41.32%	23.36%	61.59%
Lombardy	Male	LC	50.40%	50.84%	49.75%
		P	64.44%	38.04%	71.73%
		OL	20.18%	19.45%	20.02%
		Proposed - High	221.12%	31.31%	101.08%
		Proposed - Avg	70.47%	202.56%	138.76%
		Proposed - Low	1035.16%	316.99%	5339.17%
	Female	LC	49.75%	49.75%	29.36%
		P	24.74%	18.13%	24.71%
		OL	24.97%	22.81%	26.12%
		Proposed - High	88.14%	81.89%	141.31%
		Proposed - Avg	116.28%	31.01%	111.96%
		Proposed - Low	116.40%	27.27%	134.34%
Sicily	Male	LC	21.79%	19.34%	22.04%
		P	20.85%	15.13%	20.77%
		OL	16.36%	11.32%	16.54%
		Proposed - High	70.26%	36.54%	68.24%
		Proposed -Avg	150.31%	251.00%	118.06%
		Proposed - Low	34.43%	29.54%	**14.61%**
	Female	LC	8.11%	6.61%	8.37%
		P	15.98%	12.70%	15.46%
		OL	72.92%	76.62%	52.33%
		Proposed - High	61.51%	30.52%	107.60%
		Proposed -Avg	92.96%	435.95%	60.42%
		Proposed - Low	18.71%	10.02%	19.81%

Table 6.11 – The MAPE of the forecast of the models for the period 2001-2016.

Chapter 7

Conclusion

Many of the studies in the demographic literature have depicted the relationship between temperature fluctuations and mortality rates. Nevertheless, none of the existing mortality models has ever implemented a temperature-related factor, save the one proposed by Seklecka, Pantelous and O'Hare[60]. In this thesis, the emphasis of the study has been placed on the relationship between temperature changes (highest, average and lowest) and trends in mortality over the period 1974-2011, for a range of ages from 20 to 85 years, and considering both males and females from three Italian regions: Lombardy, Lazio and Sicily as proxies of the North, Centre and South of the country.

The strong negative correlations between the temperature series and both the Lee & Carter mortality index $\boldsymbol{k_t^1}$ and the age-specific central mortality rates $\boldsymbol{m_{x,t}}$ have upheld the choice of fitting and studying the temperature-related model with Italian data. Although the population size was not very large, the parametric risk was not meaningful during the fitting and the forecasting process. Furthermore, the analysis of the residuals has shown that the errors are spread around zero rather randomly. Nonetheless, the better-fitting performance of the proposed model in terms of the MAPE, compared with the Lee &Carter, Plat and O'Hare and Li models, may have resulted from over-parameterisation, since by adding penalisations for the number of parameters (AIC) the model partly loses its relative value. However, the great MAPE fitting results have been replicated during the forecasting process for only half of the datasets. This notwithstanding, the proposed model is almost always preferable

[60] Seklecka M, Pantelous AA, O'hare C. Mortality effects of temperature changes in the United Kingdom. *Journal of Forecasting*. 2017; **36**: 824-841. https://doi.org/10.1002/for.2473.

to the Lee & Carter model and may still be considered a good competitor of both the Plat model and the O'Hare and Li model for some datasets.

In conclusion, the various results among regions have once again shown that each population is different and there is not one single model which is always the best fit. Moreover, including a temperature-related factor improves the validity of this assumption for the remarkable climatic divergences worldwide.

Nevertheless, differentiating among warm months, cold months and all of the months does not substantially alter the results; therefore, it may be unnecessary to divide the year into three periods and, as a direct consequence, this model may be used accurately for the most countries. Moreover, the findings from Italy have shown that the proposed model performs better for the highest and lowest temperatures rather than for the average temperature, in accordance with all the studies which show mortality is more correlated with extreme temperatures and with similar results found from studies of the UK for cold months.

In any case, this is an absolute novelty in the literature and suggests that this analysis should be intensified in further studies.

Therefore, implementing a temperature-related factor in stochastic mortality models might lead to more painstaking fitting of mortality rates, and to better assessments of the expected value of financial and insurance products. Nevertheless, the results from the studies of the association between mortality and temperature, both in the literature and in Chapter 5 in particular, as well as those of the AIC and MAPE for the fitting process, joined with the unsteady and not excellent findings for the forecast, suggest that it may be more appropriate to implement this temperature-related factor differently; it might be considered a time-dependent factor for the (highest and lowest) temperatures, which takes into account its future fluctuations.

References

[1] Analitis A., Katsouyanni K., Biggeri A., Baccini M., Forsberg B., Bisanti L., et al. 2008. Effects of cold weather on mortality: results from 15 European cities within the PHEWE project. *American Journal of Epidemiology* **168**: 1397-1408.

[2] Booth H., Tickle L. 2008. Mortality modelling and forecasting: A review of methods. *Australian Demographic and Social Research Institute Working Paper* (3): 1-39.

[3] Brouhns N., Denuit M., Vermunt J. 2002. A Poisson log-bilinear regression approach to the construction of projected lifetables. *Insurance: Mathematics and Economics* **31**: 373-393.

[4] Brouhns N, Denuit M, Van Keilegom I. 2005. Bootstrapping the Poisson Log-Bilinear Model for Mortality Forecasting. *Scandinavian Actuarial Journal*, (3), 212-224.

[5] Cairns A.J.G., Blake D., Dowd K. 2006. A two-factor model for stochastic mortality with parameter uncertainty: Theory and Calibration. *Journal of Risk and Insurance* **73**: 687-718.

[6] Cairns, A. J. G., Blake, D., & Dowd, K. 2008. Modelling and management of mortality risk: A review. *Scandinavian Actuarial Journal*, (2-3), 79-113.

[7] Cairns A.J.G., David B., Dowd K., Coughlan G.D., Epstein D., Ong A., Balevich I. 2009. A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. *North American Actuarial Journal* **13** (1): 1-35.

[8] Cairns A.J.G., Blake D., Dowd K., Coughlan G.D., Epstein D., Khalaf-Allah

M. 2011. Mortality Density Forecasts: An Analysis of Six Stochastic Mortality Models. *Insurance: Mathematics and Economics*, **48** (3), 355-367.

[9] Continuous Mortality Investigation – Life Office Mortality Committee. Stochastic projection methodologies: Lee-Carter model features, example results and implications. 2007.

[10] Currie I.D. 2016. On Fitting Generalized Linear and non-linear Models of Mortality. *Scandinavian Actuarial Journal*, (4), 356-383.

[11] The European Parliament and of the Council of the European Union. EUR-lex. 2009. Directive 2009/138/EC. *http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2009:335:0001:0155:en:PDF*. Accessed 19 October 2017.

[12] Easterling D.R., Meehl G.A., Parmesan C., Changnon S.A., Karl T.R., Mearns L.O. 2000. Climate extremes: Observations, modeling, and impacts. *Science* **289**: 2068-2074.

[13] EIOPA, 2014. The underlying assumptions in the standard formula for the Solvency Capital Requirement calculation. *https://eiopa.europa.eu/Publications/Standards/EIOPA-14-322_Underlying_Assumptions.pdf*. Accessed 21 October 2017.

[14] Gasparrini A., Guo Y., Hashizume M., Lavigne E., Zanobetti A., Schwartz J., et al. 2015. Mortality risk attributable to high and low ambient temperature: a multi-country observational study. *Lancet* **386**: 369-375

[15] Gosling S.N., Lowe J.A., McGregor G.R., Pelling M., Malamud B.D. 2009. Associations between elevated atmospheric temperature and human mortality: a critical review of the literature. *Climatic Change* **92**: 299-341.

[16] Haberman S, Renshaw A. 2011. A Comparative Study of Parametric Mortality Projection Models. *Insurance: Mathematics and Economics* **48**, 35-55.

[17] Hajat S., Vardoulakis S., Heaviside C., Eggen B. 2014. Climate change effects on human health: projections of temperature-related mortality for the UK during the 2020s, 2050s and 2080s. *Journal of Epidemiology Community Health* **68**: 595-596.

[18] Hanewald K. 2011. Explaining mortality dynamics: the role of macroeconomic fluctuations and cause of death trends. *North American Actuarial Journal* **15**: 290-314.

[19] I.STAT. Il tuo accesso diretto alla statistica Italiana. http://dati.istat.it/Index.aspx?DataSetCode=DCIS_POPRES1#. Accessed 8 October 2017.

[20] ILMETEO.IT Archivio Meteo Italiano. https://www.ilmeteo.it/portale/archivio-meteo/. Accessed 11 October 2017.

[21] IPCC, Climate Change 2014: Impacts, Adaptation, and Vulnerability. Part A: Global and Sectoral Aspects. Contribution of Working Group II to the Fifth Assessment Report of the Intergovernmental Panel on Climate, Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

[22] Kwiatkowski D., Phillips P.C.B., Schmidt P., Shin Y. 1992. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics* **54**: 159-178.

[23] Lee R.D., Carter L. 1992. Modeling and Forecasting U.S. Mortality. *Journal of the American Statistical Association* **87**: 659-671.

[24] Levantesi S. 2017. Laboratorio di tecnica attuariale. La Sapienza università di Roma. Rome, Italy.

[25] Lovasz E. 2011. Analysis of Finnish and Swedish Mortality Data with Stochastic Mortality Models. *European Actuarial Journal*, **1** (2), 259-289.

[26] McMichael A.J., Woodruff R.E., Hales S. 2006. Climate change and human health: Present and future risks. *Lancet* **367** (9513): 859-869.

[27] Meehl G.A., Tebaldi C. 2004. More intense, more frequent, and longer lasting heat waves in the 21st century. *Science* **305**: 994-997.

[28] Minimum Capital Requirement. http://www.minimum-capital-requirement.com/. Accessed 16 October 2017.

[29] Niu G., Melenberg B. 2014. Trends in Mortality Decrease and Economic Growth. *Demography* **51**: 1755-1773.

[30] O'Hare C., Li Y. 2012. Explaining young mortality. Insurance: *Mathematics and Economics* **50**: 12-25.

[31] Patz J.A., Campbell-Lendrum D., Holloway T., Foley J.A. 2005. Impact of regional climate change on human health. *Nature* **438**: 310-317.

[32] Perron P. 1988. Trends and random walks in macroeconomic time series: Further evidence from a new approach. *Journal of Economic Dynamics and Control* **12**: 297-332.

[33] Plat R. 2009. On stochastic mortality Modelling. *Insurance: Mathematics and Economics* **45**: 393-404.

[34] Plat H.J., 2011. Essays on Valuation and Risk Management for Insurers.

Amsterdam, Netherlands. University of Amsterdam.

[35] Pitacco E., Denuit M., Haberman S., Olivieri A. 2009. Modelling longevity dynamics for pensions and annuity business. Oxford, UK. *Oxford University Press* **29**

[36] Pitacco E. 2010. Matematica e tecnica attuariale delle assicurazioni sulla durata di vita. *Lint*. Trieste, IT.

[37] Renshaw A.E., Haberman S. 2006. A cohort-based extension to the Lee-Carter model for mortality reduction factors. *Insurance: Mathematics and Economics* **38**: 556-70.

[38] Seklecka M., Pantelous A.A., O'hare C. 2017. Mortality effects of temperature changes in the United Kingdom. *Journal of Forecasting* **36**: 824-841.

[39] Solvency Capital Requirement http://www.solvency-capital-requirement.com/. Accessed 16 October 2017.

[40] Tapia Granados J.A. 2008. Macroeconomic Fluctuations and Mortality in Postwar Japan. *Demography* **45**: 323-343.

[41] Tapia Granados J.A. 2011. Mortality and Macroeconomic Fluctuations in Contemporary Sweden. *European Journal of Population* **27**(2): 157-184.

[42] Villegas A.M., Millossovich P., Kaishev V.K. 2015. StMoMo: An R Package for Stochastic Mortality Modeling. *SSRN Electronic Journal.*

[43] World Health Organization. 2014. Quantitative risk assessment of the effects of climate change on selected causes of death, 2030s and 2050s. Geneva.

Appendix A

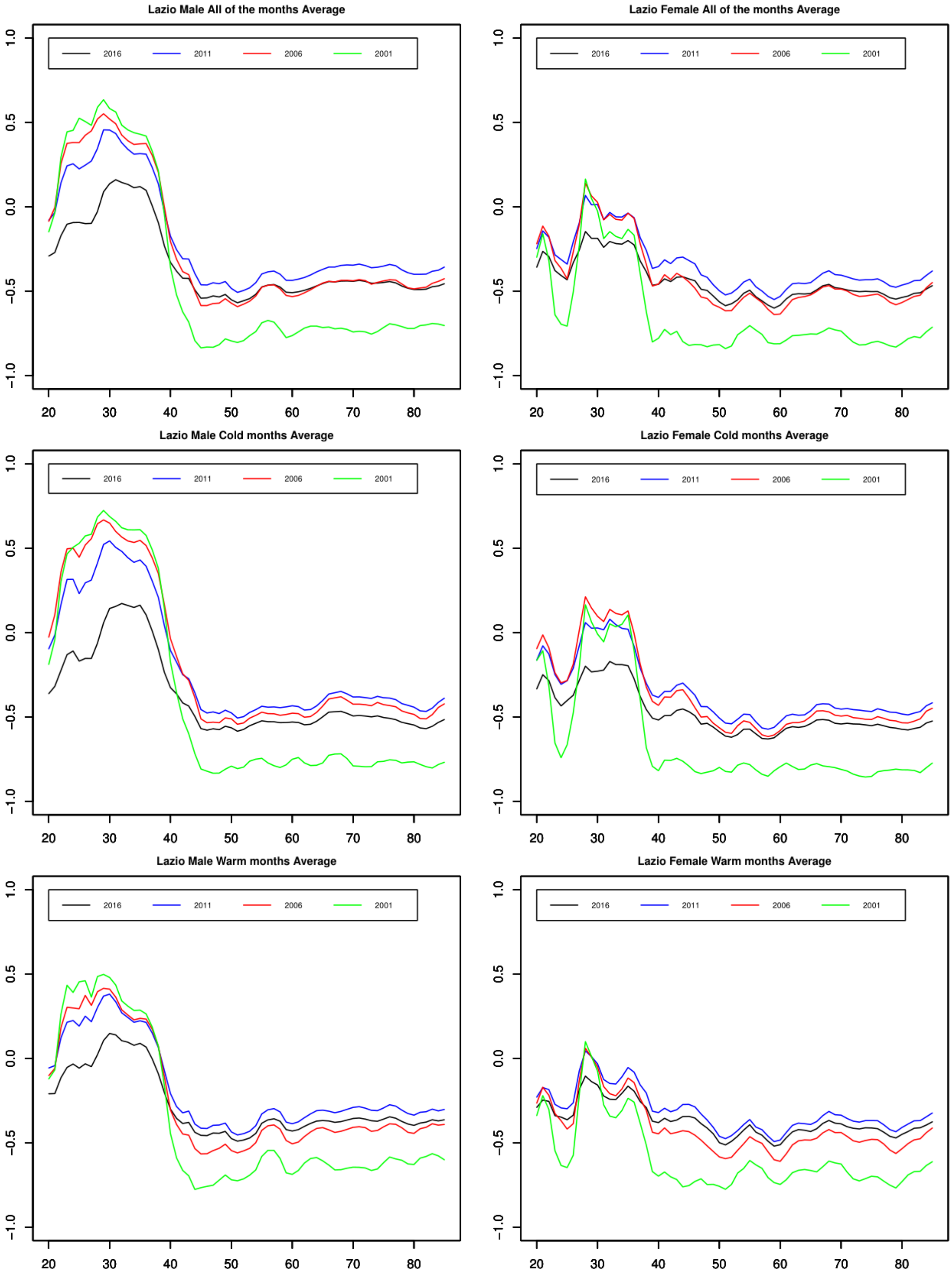

Fig A.5.7 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and average temperature for various periods of time for Lazio, by gender, from 1974 to the reported year.

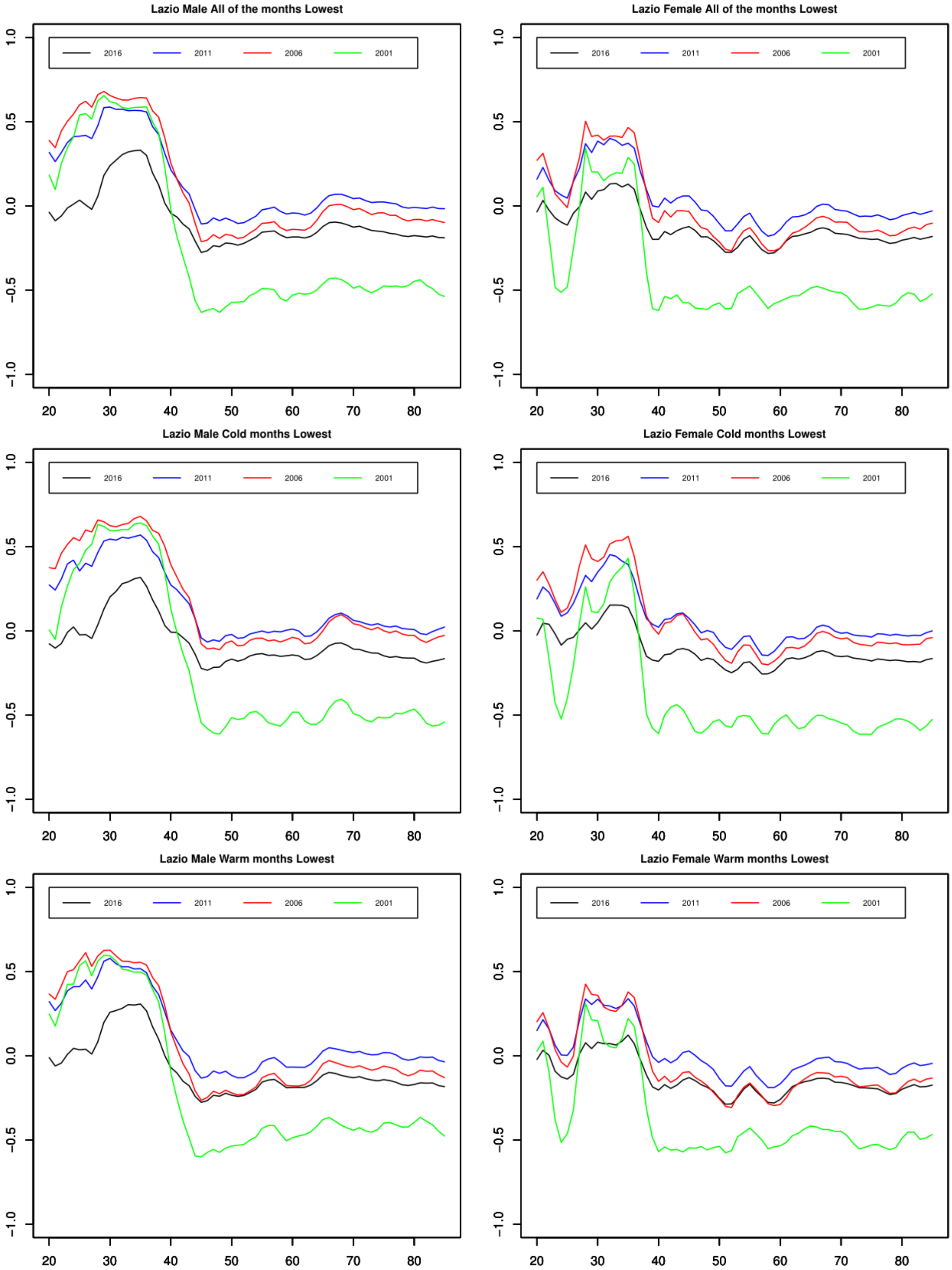

Fig A.5.8 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and lowest temperature for various periods of time for Lazio, by gender, from 1974 to the reported year.

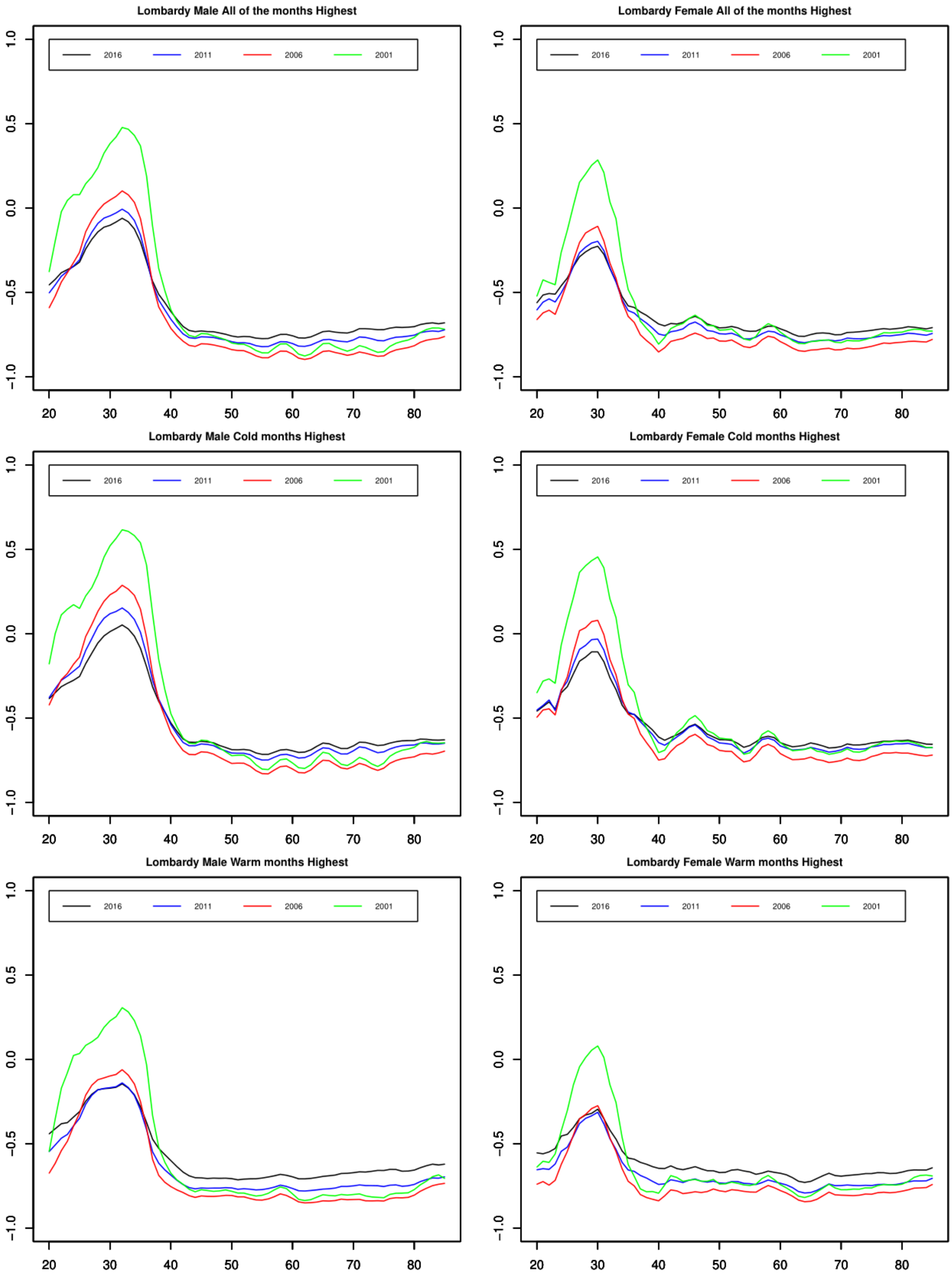

Fig A.5.9 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and highest temperature for various periods of time for Lombardy, by gender, from 1974 to the reported year.

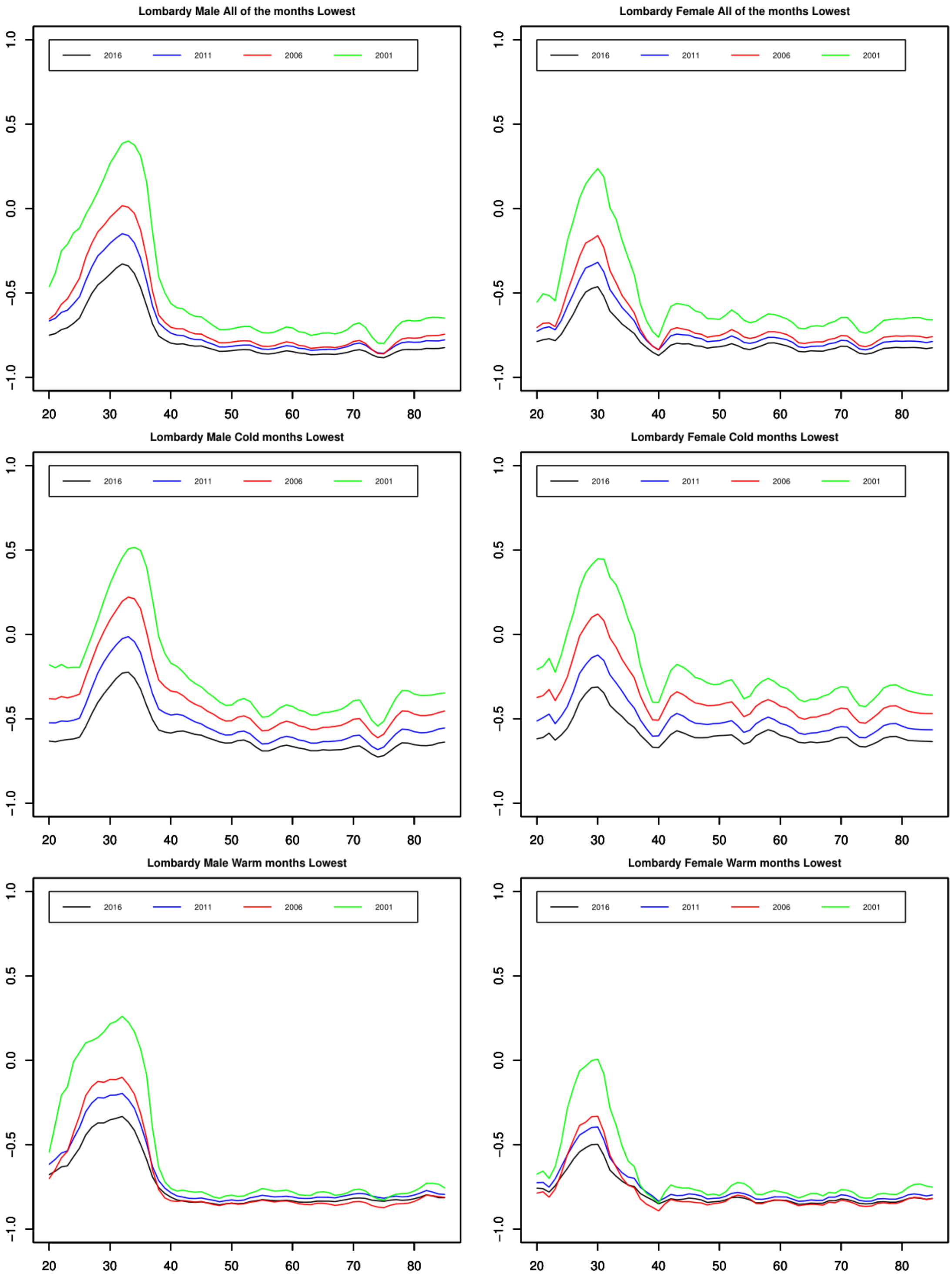

Fig A.5.10 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and lowest temperature for various periods of time for Lombardy, by gender, from 1974 to the reported year.

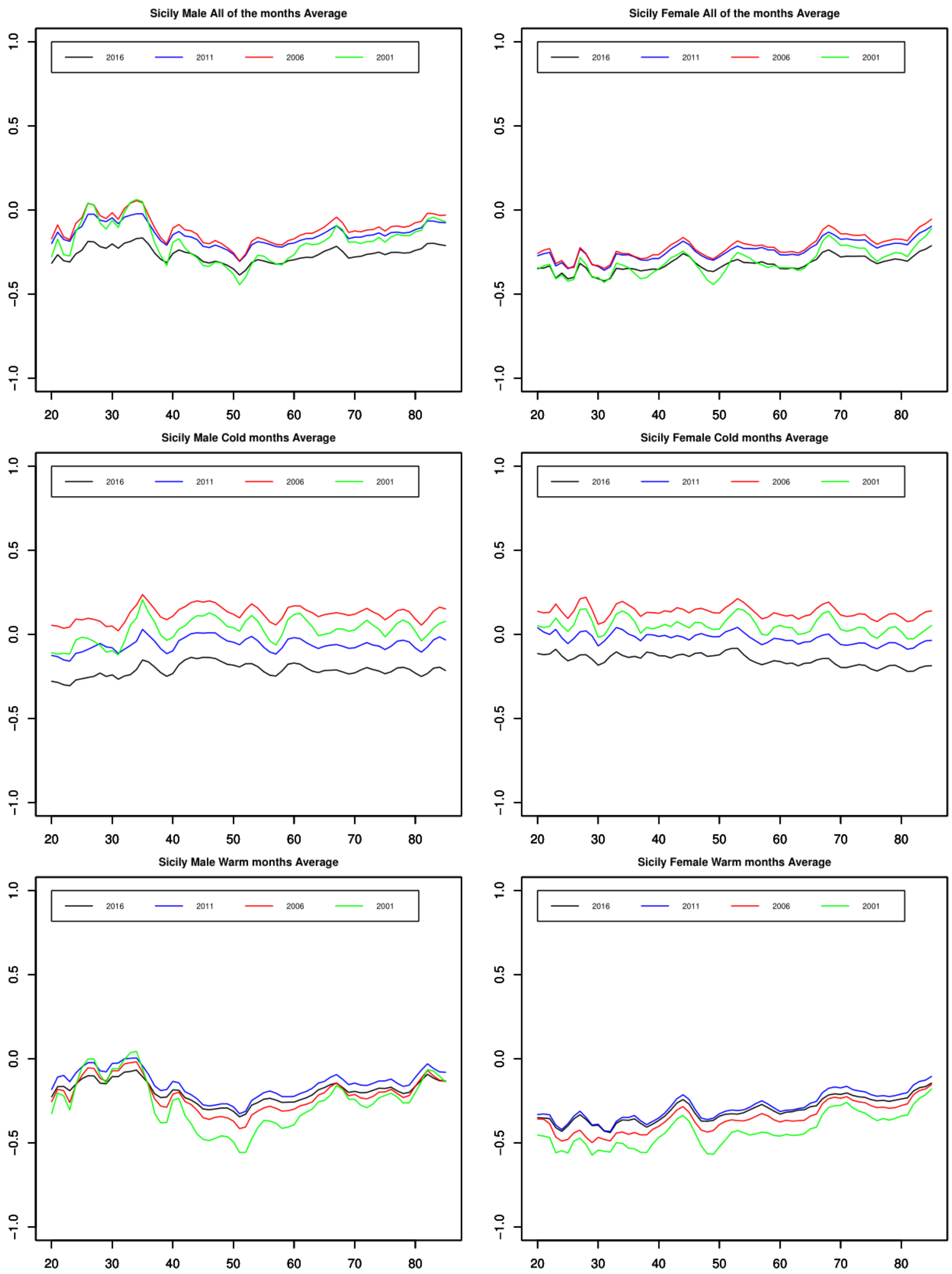

Fig A.5.11 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and average temperature for various periods of time for Sicily, by gender, from 1974 to the reported year.

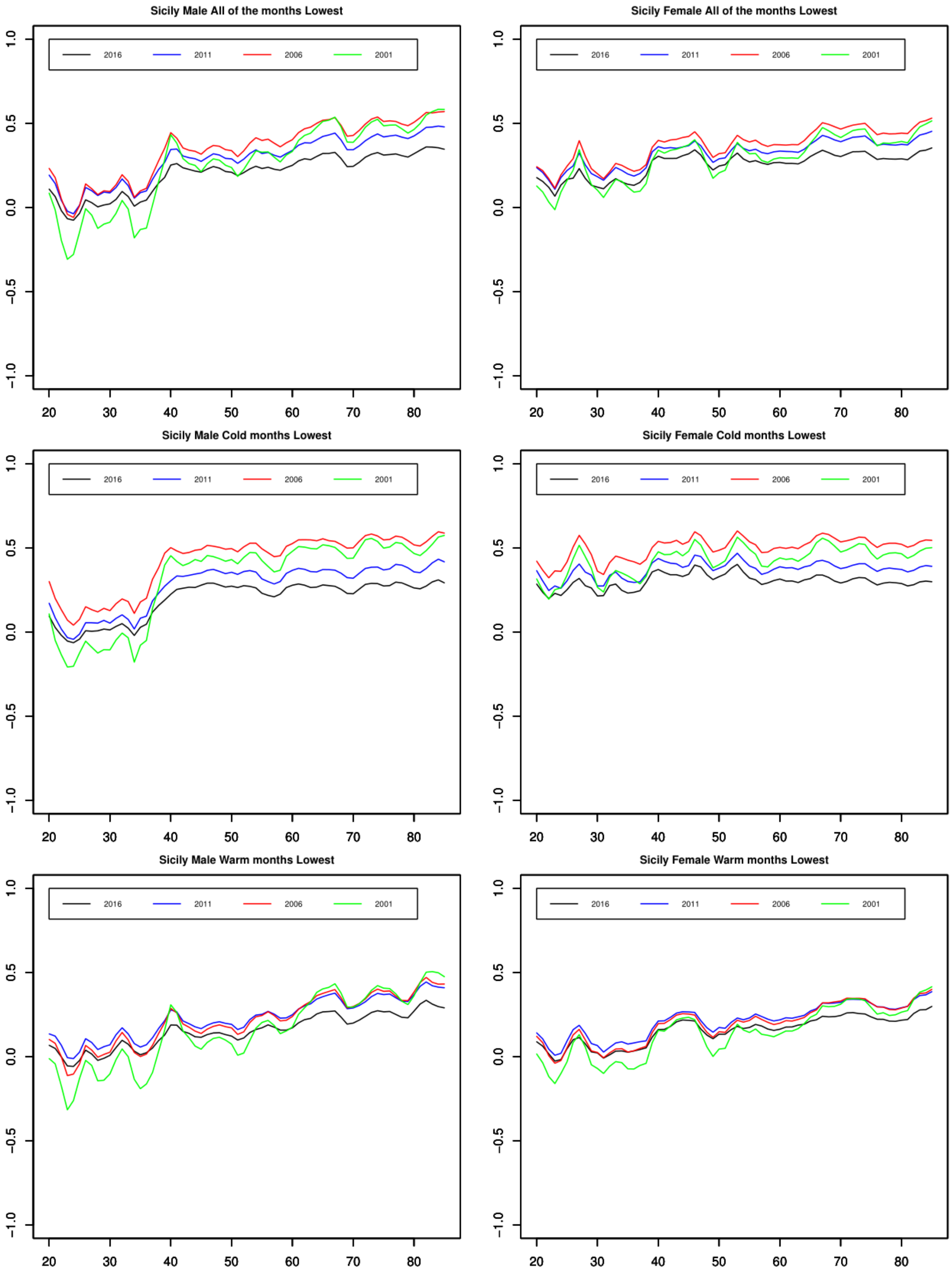

Fig A.5.12 – Robustness check of Pearson's correlation coefficient between $m_{x,t}$ and lowest temperature for various periods of time for Sicily, by gender, from 1974 to the reported year.

Appendix B

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	1.39%	1.33%	1.37%
		P	0.72%	0.64%	0.71%
		OL	0.72%	0.64%	0.70%
		Proposed - High	**0.66%**	**0.59%**	**0.65%**
		Proposed - Avg	**0.65%**	**0.57%**	**0.64%**
		Proposed - Low	**0.65%**	**0.57%**	**0.64%**
	Female	LC	1.07%	0.97%	1.06%
		P	0.73%	0.63%	0.73%
		OL	0.74%	0.64%	0.74%
		Proposed - High	**0.71%**	**0.61%**	**0.72%**
		Proposed - Avg	**0.72%**	**0.61%**	**0.72%**
		Proposed - Low	**0.70%**	**0.60%**	**0.60%**
Lombardy	Male	LC	1.69%	1.66%	1.67%
		P	0.86%	0.82%	0.85%
		OL	0.83%	0.79%	0.82%
		Proposed - High	**0.79%**	**0.72%**	**0.79%**
		Proposed - Avg	**0.78%**	**0.75%**	**0.78%**
		Proposed - Low	**0.78%**	**0.78%**	**0.78%**
	Female	LC	1.15%	1.15%	1.15%
		P	0.85%	0.60%	0.66%
		OL	0.72%	0.65%	0.65%
		Proposed - High	**0.64%**	**0.57%**	**0.64%**
		Proposed - Avg	**0.64%**	**0.57%**	**0.63%**
		Proposed - Low	**0.63%**	**0.57%**	**0.62%**
Sicily	Male	LC	1.15%	1.01%	1.07%
		P	0.67%	0.60%	0.66%
		OL	0.68%	0.60%	0.67%
		Proposed - High	**0.64%**	**0.55%**	**0.63%**
		Proposed -Avg	**0.64%**	**0.56%**	**0.63%**
		Proposed - Low	**0.61%**	**0.55%**	**0.61%**
	Female	LC	1.00%	0.89%	0.99%
		P	0.74%	0.64%	0.73%
		OL	0.76%	0.65%	0.74%
		Proposed - High	**0.71%**	**0.62%**	**0.70%**
		Proposed -Avg	**0.71%**	0.71%	**0.71%**
		Proposed - Low	**0.73%**	**0.62%**	**0.71%**

Table B.6.5 – The MAPE of the models for the period 1974-2016.

AIC	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	23021	22899	22540
		P	21078	21012	20770
		OL	21098	21022	20788
		Proposed - High	**21038**	**20989**	**20740**
		Proposed - Avg	**21011**	**20962**	**20730**
		Proposed - Low	**21033**	**20971**	**20742**
	Female	LC	19990	19824	19618
		P	19427	19358	19126
		OL	19454	19454	19150
		Proposed - High	19451	19391	19170
		Proposed - Avg	19468	19399	19182
		Proposed - Low	19490	19490	19193
Lombardy	Male	LC	29362	29151	29151
		P	24055	23960	23755
		OL	23880	23760	23593
		Proposed - High	**23839**	**23669**	**23567**
		Proposed - Avg	**23797**	**23752**	23752
		Proposed - Low	**23812**	23812	**23552**
	Female	LC	22058	21919	21648
		P	20664	20619	20387
		OL	20733	20687	20450
		Proposed - High	**20655**	**20604**	20404
		Proposed - Avg	**20647**	**20604**	**20372**
		Proposed - Low	**20645**	20629	20629
Sicily	Male	LC	21414	21260	21260
		P	20642	20642	20642
		OL	20658	20591	20316
		Proposed - High	20675	20607	20607
		Proposed -Avg	**20607**	20619	20619
		Proposed - Low	**20633**	20610	**20316**
	Female	LC	19574	19447	19153
		P	19242	19188	18882
		OL	19283	19230	18919
		Proposed - High	19265	19241	18917
		Proposed -Avg	19284	19252	19252
		Proposed - Low	19297	19254	18942

Table B.6.6 – The AIC of the models for the period 1974-2016.

MAPE	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	1.27%	1.19%	1.24%
		P	0.73%	0.65%	0.72%
		OL	0.72%	0.63%	0.70%
		Proposed - High	**0.65%**	**0.57%**	**0.63%**
		Proposed - Avg	**0.63%**	**0.55%**	**0.62%**
		Proposed - Low	**0.64%**	**0.56%**	**0.63%**
	Female	LC	1.02%	0.93%	1.02%
		P	0.69%	0.59%	0.69%
		OL	0.71%	0.61%	0.70%
		Proposed - High	**0.66%**	**0.57%**	**0.57%**
		Proposed - Avg	**0.67%**	**0.57%**	**0.67%**
		Proposed - Low	**0.66%**	**0.56%**	**0.66%**
Lombardy	Male	LC	1.56%	1.51%	1.54%
		P	0.80%	0.75%	0.79%
		OL	0.75%	0.71%	0.74%
		Proposed - High	**0.70%**	**0.63%**	**0.69%**
		Proposed - Avg	**0.70%**	**0.63%**	**0.69%**
		Proposed - Low	**0.70%**	**0.62%**	**0.69%**
	Female	LC	1.09%	1.02%	1.07%
		P	0.62%	0.56%	0.62%
		OL	0.65%	0.58%	0.64%
		Proposed - High	**0.57%**	**0.51%**	**0.56%**
		Proposed - Avg	**0.57%**	**0.51%**	**0.56%**
		Proposed - Low	**0.57%**	**0.52%**	**0.56%**
Sicily	Male	LC	1.11%	1.03%	1.09%
		P	0.67%	0.59%	0.65%
		OL	0.65%	0.57%	0.64%
		Proposed - High	**0.62%**	0.62%	**0.61%**
		Proposed -Avg	**0.62%**	**0.54%**	**0.61%**
		Proposed - Low	**0.61%**	**0.54%**	**0.54%**
	Female	LC	1.01%	0.90%	0.99%
		P	0.72%	0.62%	0.71%
		OL	0.74%	0.64%	0.73%
		Proposed - High	**0.70%**	**0.61%**	**0.70%**
		Proposed -Avg	**0.70%**	**0.60%**	**0.69%**
		Proposed - Low	**0.70%**	**0.60%**	**0.70%**

Table B.6.7 – The MAPE of the models for the period 1974-2006.

AIC	Gender	Model	All of the Months	Cold Months	Warm Months
Lazio	Male	LC	17612	17467	17254
		P	16353	16286	16109
		OL	16336	16265	16095
		Proposed - High	**16303**	**16249**	**16063**
		Proposed - Avg	**16279**	**16224**	**16053**
		Proposed - Low	**16297**	**16234**	**16065**
	Female	LC	15458	15325	15325
		P	15038	14995	14808
		OL	16095	15017	14833
		Proposed - High	15040	15013	14825
		Proposed - Avg	15056	15022	14835
		Proposed - Low	15079	15042	14849
Lombardy	Male	LC	22538	22274	22059
		P	18531	18436	18312
		OL	18419	18318	18318
		Proposed - High	**18303**	**18184**	**18103**
		Proposed - Avg	**18303**	**18173**	**18113**
		Proposed - Low	**18293**	**18293**	**18103**
	Female	LC	17305	17220	16985
		P	16155	16151	15946
		OL	16185	16177	15975
		Proposed - High	**16128**	**16143**	**15923**
		Proposed - Avg	**16129**	16152	**15921**
		Proposed - Low	**16129**	16169	**15918**
Sicily	Male	LC	16675	16548	16335
		P	16014	15967	15754
		OL	16008	15956	15748
		Proposed - High	**16002**	15975	15758
		Proposed -Avg	16032	15989	15770
		Proposed - Low	**16000**	15976	15755
	Female	LC	15257	15165	14927
		P	14949	14915	14915
		OL	14971	14934	14694
		Proposed - High	14966	14956	14702
		Proposed -Avg	14991	14970	14718
		Proposed - Low	14983	14957	14719

Table B.6.8 – The AIC of the models for the period 1974-2006.